Robust Linear Parameter-Varying State-Observer for an Induction Motor

Najmeh Eghbal*, Nader Fasanghari, Samira Rafiee*****
*** Islamic Azad University, Sabzevar Branch, Iran, najmeh.eghbal@gmail.com**
**** Islamic Azad University, Sabzevar Branch, Iran, nader_fasanghari@yahoo.com**
*****Khorasan Regional Electrical Company, samirafiee@gmail.com**

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Abstract

This paper addresses observation and estimation of some states of an induction motor with parameter uncertainty. State observation is an important step in controller synthesis for systems which measurement of all states is not possible or affordable. At the first, a linear parameter varying (LPV) model is presented for induction motor and then via extending LPV observers for uncertain systems, a robust LPV observer is designed for induction motor. In the proposed method, the immeasurable parameters are considered as uncertainty. The results of simulation verify the validation of designed observer.

Key words: observer; Linear Parameter Varying (LPV); parameter uncertainty; induction motor.

1. Introduction

An induction motor is a type of alternating current motor that widely used in industrial applications because of high power, simple structure, low cost, ease of maintenance and its ability to speed controlling [1-3]. There are several methods for controlling induction motors but most of synthesizing methods are based on linearization around nominal point [4, 5] or nonlinear control. Linearization of the model is only valid over small region around nominal point. Because of inherent nonlinearity of the induction motor and its parameter variation over wide range by changing the temperature, linearization based methods are not suitable. Two nonlinear methods for controller synthesizing are sliding mode and feedback linearization [7, 8] which are more complex than linear methods and this complexity made them hard to apply. One of the most efficient synthesizing methods for nonlinear systems is gain scheduling. Gain scheduling technique is a method that despite using linear tools, keeps nonlinear description of the system. It is the best choice for nonlinear systems that have parameters which change the nominal conditions. Up to now various gain scheduling methods are proposed for nonlinear systems with varying parameters. These methods design different controllers with

uniform structure in different points of parameter space for the system, then based on the value of parameters and via an interpolation law, the global controller is synthesized based on the designed local controllers. Introducing gain scheduling problem in the form of LPV systems made this problem easy to solve and strong in the theoretical point of view [9]. Gain scheduling based on LPV systems is a powerful method for controlling nonlinear systems with parameter uncertainty. Before designing the controller, the nonlinear parameter varying system must be modeled in LPV form. In [11] and [12] some LPV modeling methods are presented. In [13] LPV model of induction motor after linearization obtained which is true around nominal point. In [14] LPV model by considering nonlinear behavior of system has been presented without uncertainty. Sometimes LPV model of a system has many parameters. It is obvious that measuring all of these parameters is necessary but it has two problems: 1- high cost of measuring these parameters and also the synthesis method, 2- some parameters are not available and measurable. Induction motor modeling has both of these problems. Since magnetic flux is not measurable for controlling induction motor based on LPV controllers, it is necessary to estimate states. It was shown that for LPV systems, LPV observers have less conservativeness with respect to general observers [13, 14].
In this paper the LPV modeling of induction motor is addressed. Since some parameters of the system are not available for measurement, they considered as uncertainty in the LPV model. Then based on the resulted LPV model, immeasurable states of system are estimated. This paper combines the ideas of LPV observers and robust observers to design a robust LPV observer. In II section, synthesizing LPV observer is explained. Section III included the main result of this paper on robust LPV observer. LPV modeling of induction motor is presented in section IV. In section V, numerical example is given to demonstrate the effectiveness of the theoretical results. And finally, conclusions are drawn in section VI.

2.LPV Observer

Linear parameter varying systems are described by:

$$\begin{cases} \dot{x} = A(\theta)x + B(\theta)u \\ y = C(\theta)x \end{cases} \tag{1}$$

where $\theta(t) \in \Theta$ is time-varying parameter vector:

$$\Theta = \left\{ \theta \middle| \theta = [\theta_1 \quad \dots \quad \theta_m]^T, \ \underline{\theta}_i \le \theta_i \le \bar{\theta}_i, \ i = 1, \dots, m \right\}$$

Thus Θ is defined as a convex polytopic and it could be expressed based on its vertices as:

$$\Theta = Co\{w_1, \dots, w_r\} = \left\{ \sum_{i=1}^{r} \alpha_i w_i, \ \alpha_i \ge 0, \ \sum_{i=1}^{r} \alpha_i = 1 \right\}$$

where $w_i, i = 1, \dots, r$ are vertices of Θ and Co is an operator which results all linear combination of operands. For LPV systems which their state matrices are dependent to parameters affinely, we have:

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & 0 \end{bmatrix} = Co\left\{ \begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix}_{i=1,\dots,r} \right\}$$

where

$$\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} = \begin{bmatrix} A(w_i) & B(w_i) \\ C(w_i) & 0 \end{bmatrix}$$

Based on Luenberger method, the observer for LPV system (1) can be designed as [15]:

$$\dot{\hat{x}}(t) = A(\theta)\hat{x}(t) + B(\theta)u(t) + L(y(t) - C(\theta)\hat{x}(t))$$

where $\hat{x}(t)$ is the vector of estimated states and L is the gain of observer.

For computing observer gain, first the estimation error is defined as:

$$\varepsilon = x - \hat{x}$$

The state equation of ε is then expressed as:

$$\dot{\varepsilon} = (A - LC)\varepsilon \tag{2}$$

Asymptotic stability of (2) means convergence of estimated states to real ones. For stability of system (2), consider Lyapunov function $V = \varepsilon^T P \varepsilon$ where P is a positive definite symmetric matrix. The sufficient condition for asymptotically stability of (2) is $\dot{V} \prec 0$ which is equivalent to the following inequality:

$$(A(\theta) - LC(\theta))^T P + P(A(\theta) - LC(\theta)) \prec 0$$

By defining $X = PL$, the above inequality can be written as:

$$A^T(\theta)P - C^T(\theta)X^T + PA(\theta) - XC(\theta) \prec 0 \tag{3}$$

The inequality mentioned above must be true for all possible trajectories of $\theta(t)$. This is a problem with infinite number of constraints, but because of affine parameter dependency of the LPV system, it is sufficient that the above inequality holds only for vertices of the convex polytopic Θ. So (3) is equivalent to:

$$A_i^T P - C_i^T X^T + PA_i - XC_i \prec 0, \quad i = 1, \ldots, r$$

and L can be obtained from :

$$L = P^{-1}X$$

where P and X will be the solution of the following feasibility problem which presented in a set of linear matrix inequalities (LMI):

$$\begin{cases} A_i^T P - C_i^T X^T + PA_i - XC_i \prec 0, \quad i = 1, \ldots, r \\ P \succ o \end{cases}$$

If θ is measurable, then by designing local observer for every vertices of Θ, observer can be designed for system by linear combination of local observers.

For designing LPV observer, following feasibility problem must be solved:

$$\begin{cases} A_i^T P - C_i^T X_i^T + PA_i - X_i C_i \prec 0, \quad i = 1, \ldots, r \\ P \succ o \end{cases}$$

The gains of local observers are obtained from $L_i = P^{-1}X_i$. So as mentioned, LPV observer is linear combination of local observers:

$$L = \sum_{i=1}^{r} \alpha_i L_i$$

where α_i are variables that in any time they satisfy in the following equations:

$$\theta(t) = \sum_{i=1}^{r} \alpha_i(t) w_i,\ \alpha_i(t) \geq 0,\ \sum_{i=1}^{r} \alpha_i(t) = 1$$

In the next part, the method proposed in this paper will be described.

3. Robust LPV Observer

Now consider that some components of θ are measurable and the others which are immeasurable are considered as parameter uncertainties. The only information that exists about parameter uncertainties is the range of their variations. In this way parameter vector θ is defined as:

$$\theta = \begin{bmatrix} \theta_m^T & \theta_{um}^T \end{bmatrix}^T$$

where θ_m is measurable parameter vector and θ_{um} is immeasurable parameter vector. If $w_i^m,\ i = 1,\ldots,r_m$ are vertices that define the polytopic which θ_m belongs to it and $w_j^{um},\ j = 1,\ldots,r_{um}$ are vertices that define the polytopic which θ_{um} belongs to, then by defining $w_{ij} = \begin{bmatrix} w_i^{m^T} & w_j^{um^T} \end{bmatrix}^T$ the problem of synthesizing robust LPV observer for uncertain LPV system could be formulated as following.

Synthesizing robust LPV observer leads to the following set of LMIs:

$$\begin{cases} A^T(w_{ij})P - C^T(w_{ij})X_i^T + PA(w_{ij}) - X_iC(w_{ij}) \prec 0, \\ \qquad\qquad\qquad\qquad i = 1,\ldots,r_m,\ j = 1,\ldots,r_{um} \\ P \succ o \end{cases} \tag{4}$$

By measuring θ_m and calculating α_i from:

$$\theta_m(t) = \sum_{i=1}^{r_m} \alpha_i(t) w_i^m,\ \alpha_i(t) \geq 0,\ \sum_{i=1}^{r_m} \alpha_i(t) = 1$$

the gain of the robust polytopic observer is given by:

$$L = \sum_{i=1}^{r_m} \alpha_i L_i$$

4. LPV Modeling of Induction Motor

Nonlinear equation of induction motor is:

$$\begin{aligned}
d\omega/dt &= n_p(M/JL_R)(i_{Sb}\psi_{Ra} - i_{Sa}\psi_{Rb}) - \tau_L/J \\
d\psi_{Ra}/dt &= -(R_R/L_R)\psi_{Ra} - n_p\omega\psi_{Rb} + M(R_R/L_R)i_{Sa} \\
d\psi_{Rb}/dt &= -(R_R/L_R)\psi_{Rb} + n_p\omega\psi_{Ra} + M(R_R/L_R)i_{Sb} \\
u_{Sa} &= R_S i_{Sa} + \sigma L_S di_{Sa}/dt + (M/L_R)d\psi_{Ra}/dt \\
u_{Sb} &= R_S i_{Sb} + \sigma L_S di_{Sb}/dt + (M/L_R)d\psi_{Rb}/dt
\end{aligned} \tag{5}$$

where n_p is number of rotor poles, J is rotor inertia moment, τ_L is the torque constant, M is mutual inductance, R_S and R_R are stator and rotor resistances, L_S and L_R are stator and rotor inductances, i_{sa} and i_{sb} are two components of the stator current, ψ_{Ra} and ψ_{Rb} are two components of the rotor flux, u_{sa} and u_{sb} two components of the stator voltage and ω is the rotor speed. Also σ is the leakage coefficient of Blondel:

$$\sigma = 1 - M^2/L_R L_S$$

System (5) can rewrite in state space equation form:

$$\begin{aligned}
d\omega/dt &= \mu(i_{Sb}\psi_{Ra} - i_{Sa}\psi_{Rb}) - \tau_L/J \\
d\psi_{Ra}/dt &= -\eta\psi_{Ra} - n_p\omega\psi_{Rb} + \eta M i_{Sa} \\
d\psi_{Rb}/dt &= -\eta\psi_{Rb} + n_p\omega\psi_{Ra} + \eta M i_{Sb} \\
di_{Sa}/dt &= \eta\beta\psi_{Ra} + \beta n_p\omega\psi_{Rb} - \gamma i_{Sa} + u_{Sa}/\sigma L_S \\
di_{Sb}/dt &= \eta\beta\psi_{Rb} - \beta n_p\omega\psi_{Ra} - \gamma i_{Sb} + u_{Sb}/\sigma L_S
\end{aligned} \tag{6}$$

where

$$\mu = n_p M/JL_R$$

$$\eta = R_R/L_R$$

$$\beta = M/\sigma L_R L_S$$

$$\gamma = M^2 R_R/\sigma L_R^2 L_S + R_S/\sigma L_S$$

If consider ω as measurable parameter and R_R, R_S as uncertain immeasurable parameters of the system, then state-space matrices of LPV system can be obtained as:

$$x = \begin{bmatrix} i_{Sa} & i_{Sb} & \psi_{Ra} & \psi_{Rb} \end{bmatrix}^T$$

$$u = \begin{bmatrix} u_{Sa} & u_{Sb} \end{bmatrix}^T$$

$$y = \begin{bmatrix} i_{Sa} & i_{Sb} \end{bmatrix}^T$$

$$\theta = \begin{bmatrix} \omega & \eta & \gamma \end{bmatrix}$$

$$A = \begin{bmatrix} -\gamma & 0 & \eta\beta & \beta n_p\omega \\ 0 & -\gamma & -\beta n_p\omega & \eta\beta \\ \eta M & 0 & -\eta & -n_p\omega \\ 0 & \eta M & n_p\omega & -\eta \end{bmatrix}$$

$$B=\begin{bmatrix} 1/\sigma L_S & 0 \\ 0 & 1/\sigma L_S \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\quad C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It is noted that since η and γ are functions of R_R and R_S, so η and γ can be selected instead of R_R and R_S as uncertain immeasurable parameters.

5. Simulation Results

Characteristics of induction motor are listed in table I. In this system parameters change as following:

$$\begin{gathered} -200 \ \text{rad/sec} \prec \omega \prec 200 \ \text{rad/sec} \\ 0.0935 \ \Omega \prec \mathrm{R_R} \prec 0.2805 \ \Omega \\ 0.131 \ \Omega \prec \mathrm{R_S} \prec 0.393 \ \Omega \end{gathered}$$

symbol	value
L_S	$146.5 \quad mH$
L_R	$146.5 \quad mH$
M	$143 \quad mH$
J	$11.06 \quad kg-m^2$
n_p	2
τ_L	$2000 \quad Nm$
R_S	$0.1965 \ \Omega$
R_R	$0.1402 \ \Omega$

TABLE I: characteristics of induction motor

According to this specifications and solving LMI problem in (4), variables obtained as:

$$P=\begin{bmatrix} 0.0021 & 0 & 0.0079 & 0 \\ 0 & 0.0021 & 0 & 0.0079 \\ 0.0079 & 0 & 2.3536 & 0 \\ 0 & 0.0079 & 0 & 2.3536 \end{bmatrix}$$

$$X_1=\begin{bmatrix} 1.6357 & -0.0048 \\ 0.0048 & 1.6357 \\ 0.0716 & 11.2788 \\ -11.2788 & 0.0716 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 1.6413 & 0.0136 \\ -0.0136 & 1.6413 \\ 0.1307 & -112.7933 \\ 112.7933 & 0.1307 \end{bmatrix}$$

Since $L_i = P^{-1}X_i$, we have:

$$L_1 = \begin{bmatrix} 806.6519 & -21.0063 \\ 21.0063 & 806.6519 \\ -2.6724 & 4.8599 \\ -4.8599 & -2.6724 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 809.4764 & 193.0897 \\ -193.0897 & 809.4764 \\ -2.6561 & -48.5723 \\ 48.5723 & -2.6561 \end{bmatrix}$$

Fig. 1 and Fig. 2 show simulation results for real and estimated states. Despite of difference in initial conditions of real and estimated states, designed observer has appropriate ability to estimate immeasurable states. After little time, the convergence of estimated states to real ones could be seen in Fig. 1 and Fig. 2. LPV model of induction motor which defined in this paper is one by one description of nonlinear system to LPV model, so the synthesizing was done without omission nonlinear behavior of original system and it is valid in all parameter space and state space.

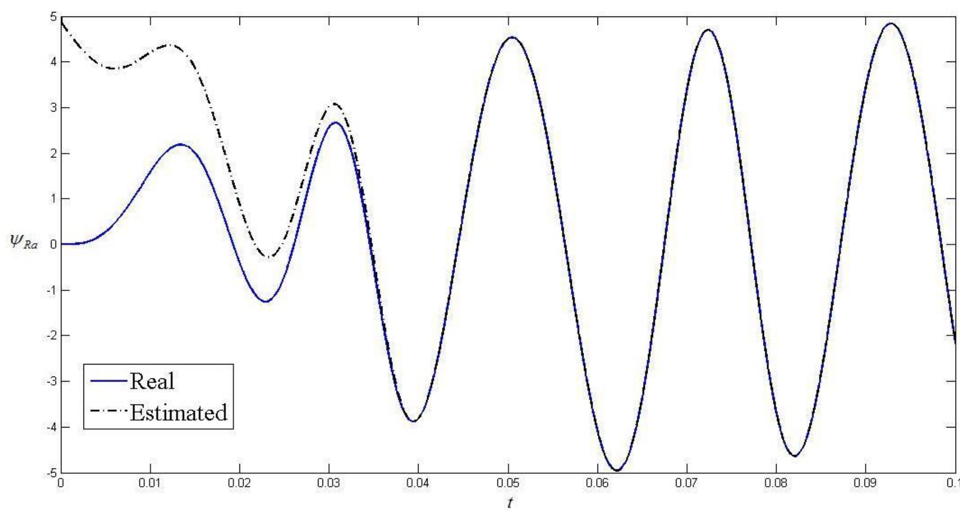

Fig. 1: Real and estimated $\psi_{Ra}(t)$

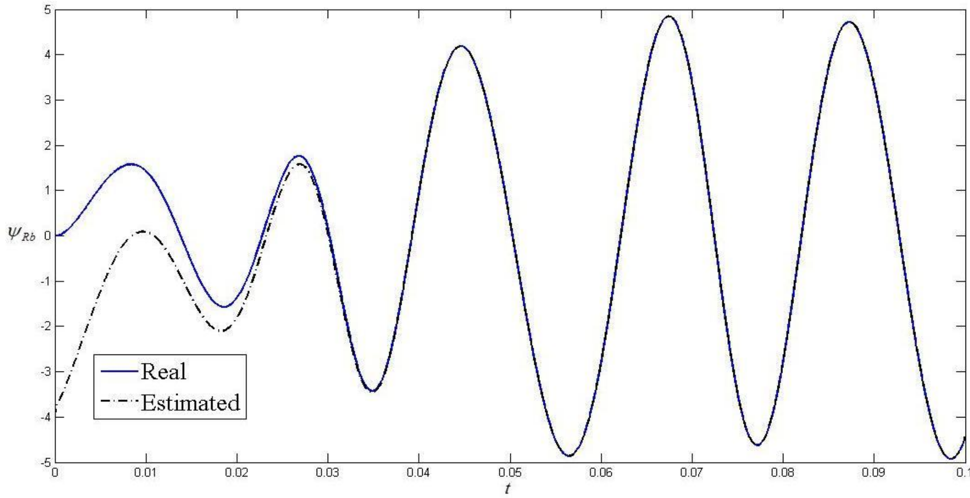

Fig. 2: Real and estimated $\psi_{Rb}(t)$

6. Conclusions

Rotor flux of induction motor is immeasurable, so in this paper, based on LPV observer, state estimation method has been addressed. By considering motor speed as measurable parameter, LPV observer designed. As stator and rotor resistances change when temperature varies, it is necessary to consider this uncertainty in synthesizing. In this paper by designing robust observer for induction motor, acceptable estimation for rotor flux has been obtained.

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