Satellite Attitude Determination Using GPS Receiver Based on Wahba Cost Function

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Abstract

After launching and injecting a satellite into the orbit, attitude determination and control is one of the most important tasks for the satellite to stay in its orbit. This is done by different methods having different accuracies. Some examples for attitude determination and control techniques are “spin stabilized”, “3 axis stabilization”, “gravity gradient method”, “using 3 gyros in 3 directions”, etc.
GPS is usually used in satellites to determine the satellite altitude and orbit parameters. It is used as a complementary for the attitude determination sensors. Attitude determination is critical for modern spacecrafts. An ordinary attitude determination system is composed of some rather cheap magnetometers, sun sensors, star trackers or earth horizon gyros. GPS navigation system can be valuable and effective for these satellite sensors. It can also determine the satellite attitude solely.
Here in this paper a method is proposed for attitude determination using GPS. This Method is based on the carrier phase of a received signal from two or more antennas mounted on the satellite’s body. Phase difference between antennas can be used to determine the attitude and direction of the satellite. The rate of phase change shows the rate of attitude change.
Based on the theories, this method would have a high reliability and it can be used as an attitude determination technique. The number of GPS Antennas should be calculated to achieve the best accuracy. Its precision is acceptable and in some cases even higher in respect of other methods.

Key words: Attitude Determination, GPS, Satellite, Wahba cost function

1. Introduction

Attitude of a satellite means its direction in the space. Automatic attitude determination is necessary for modern spacecraft. Basically motion of a satellite is described by its position, velocity, attitude and relative motion. Attitude assessment consists of attitude determination, controlling and next motion prediction for satellite. Attitude determination contains attitude stabilization and controlling the attitude maneuver. The authors aim is to present a method for attitude determination using GPS with a special layout of antennas with a good precision for satellite attitude determination.

2.Satellite attitude determination methods

Earth, sun and stars are usually considered as a source or reference for attitude determination. The precision depends on the hardware and process algorithm. An ordinary attitude determination system contains some relatively cheap magnetometers, sun sensors, star trackers or Gyros (earth horizon detector). While GPS navigation system can be a valuable and effective complementary system for the sensors. It can also be solely used as an attitude determination system [4]. For LEO satellites a combination of two sensors (sun sensor and earth sensor) are used for attitude determination in 3 axes. Stars are used for very precise measurements (about some arc seconds). In table (1) there is a comparison between different attitude determination methods.

Method of Attitude determination	**Precision (deg)**	**Process**
IMU (Inertial Measurement Unit)	0.1	3 Heavy and complicated Gyros are used to determine the angular velocity and 3 accelerometers are used to determine the linear acceleration [10].
Earth sensor *(especially infrared sensor)*	0.1 for LEO and 0.02 for GEO	Comparing between earth atmosphere heat and deep space cold, shows the infrared waves. So roll and pitch axes attitude can be determined [10,1]. This sensor has a variety of equipment.[1,7]
Sun sensor	Better than 0.1	Measuring sun vision angle, satellite attitude is determined (usually for yaw axis) [10,9,7].
Star sensor	0.01	This sensor senses light intensity and star size and tracks a special star [10,1]. This sensor is heavy with a high energy consumption [9,7]. The map of the sky is installed in the memory of the sensor.
Velocity sensor (Gyro)	1 – 100 deg./sec	This sensor measures the variations of direction as velocity or displacement. The main advantage of these sensors is satellite angular determination without need to space angle determination [9,7].
Magnetometer	0.5	This sensor measures the satellite attitude by comparing local magnetic field with the values in it the memory (only for LEO). They have less weight and less power consumption with a higher Reliability [10].
GPS receiver	To be discussed	Using phase difference between received signals by different antennas.

Table (1): Comparison between different attitude determination methods

3- Theorem of "Attitude determination using GPS"

Nowadays GPS is used in the spacecraft for many purposes. Orbit and attitude determination, locating the spacecrafts in the space, launcher trajectory assessment and also time synchronization are some of the applications. Attitude determination using GPS is done based on the carrier Phase of a signals received from two or more antennas mounted on the satellite body. Phase difference between antennas can be the determinant of attitude and orientation of the spacecraft. The variation rate of these phases gives the rate of Attitude variation.

Practically two receivers is used on the satellite (or any other object having GPS) to receive the signals from two antennas mounted on the satellite body. They have a known distance from each other. Indeed these signals are from a GPS satellite. The phase difference is

calculated based on received signals and this is directly related to satellite Attitude (more than two antennas is necessary for the system to work continuously) [9].
The antenna is not always directed to the sky (sometimes directed to the earth), therefore a unique antenna can't be used to receive the signals in all directions. Two omnidirectional antennas are installed on right and left or up and down sides of the satellite. Having two antennas will cause a Parasite region because the signals are directly mixed. Parasite zone ranges ±15 degrees around the symmetry plane of the antennas. The GPS receiver can't receive the signals in parasite zone. This causes a reduced precision and lack of reliability in attitude determination. This zone is rotating as the satellite rotates around the earth. Parasite zone causes the receiver to recognize less GPS satellites.
Under these circumstances, GPS receivers can be employed as master and slave. They receive the signals from GPS antennas independently and demodulate them. Hence the signals from two antennas will not be mixed and coverage area will be whole the sky. If the Antennas are mounted in a proper position, the GPS satellites will be always visible during the flight [6].
Precision of GPS is limited by distance of antennas, rigidity of antenna bases and phased noise multipath in receiver. In the method "Attitude determination using GPS satellites", carrier phase difference measurement is used and this causes the S/A (Selective Availability) error to be small (if existing) with no reduction in precision. GPS precision is restricted firstly by design parameters of the antenna structure and also receiver electronics limitations [5].
A precision better than 0.1 degrees is achievable by employing the best technologies. Necessary precision tolerance for satellite attitude determination is 5 degrees for simple satellites and better than 3×10^{-6} degrees for spacecrafts like Hubble telescope. A precision about 3×10^{-6} degrees is achievable by using star sensors. In this case GPS has a lower precision and can't replace the star sensor, but it is a good choice for most of mission requirements. Precision of this attitude determination method is mentioned in different references. Reference [8] notices the precision of about 0.4 up to 0.2 degrees for GPS. Reference [5] announces the precision of less than 1 minute using GLONASS system.

3-1- Describing "Attitude determination using GPS method" by mathematic equations

This method is to determine the attitude of a subject in the space in reference coordination [5]. Two GPS signal receivers (GPS antenna) are positioned in two points of a moving object (which should be attitude determined) with a distance of "d" (points A and B in Fig.1).

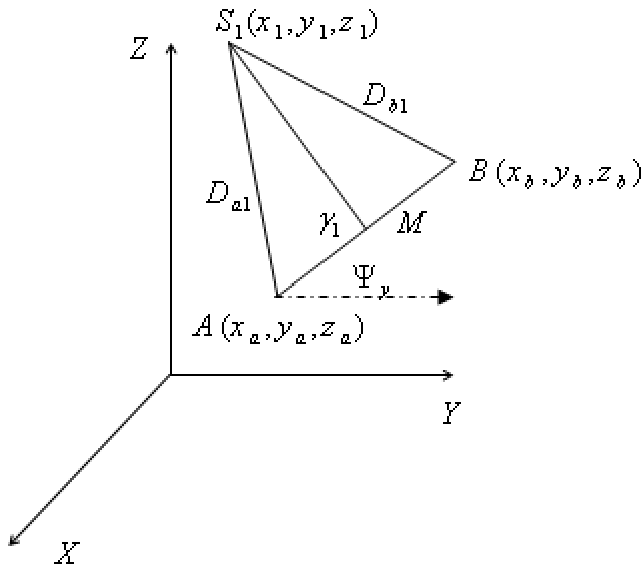

Fig. (1): status of antennas and GPS satellite

The position of AB line is known in relation to the object (coordination system integrated to the object). By determining situation of line "AB", attitude of object (Euler triple angles) in a reference coordination system can be determined. Status of line "AB" is determined by ($\cos\psi_x$, $\cos\psi_y$, $\cos\psi_z$) and also the relation to OXYZ geocentric coordination system. Assume receivers in A and B measure their distance from NS1 (Navigation Satellite) as D_{a1} , D_{b1}. Coordination of NS1 is (X1, Y1, Z1). Then the phase difference between A and B is:

(1) $$\Delta\phi_1 = 2\pi \frac{Da_1 - Db_1}{\lambda}$$

When λ is wavelength of the signal sent from NS_1. γ_1 is the angle between AB vector and S_1M vector which connects center of line AB to NS_1. The relation between $\Delta\phi_1$ and γ_1 is:

(2) $$\cos\gamma_1 = \Delta\phi_1 \frac{\lambda}{2\pi d}$$

The relation between angle and cosines vector of line AB is as below:

(3) $$\cos\gamma_1 = \mu_{x1}\cos\psi_x + \mu_{y1}\cos\psi_y + \mu_{z1}\cos\psi_z$$

Where μ_{x1}, μ_{y1} and μ_{z1} are coefficients calculated by equations containing coordinates of S_1 and M by measuring D_{a1}, D_{b1}. Equation (3) contains 3 unknowns ($\cos\psi_x$, $\cos\psi_y$, $\cos\psi_z$). Hence we need 3 equations similar to equations (1) and (2) which obtained by A and B receivers (using signals from two other GPS satellites in different positions). Now we have three independent equations. Assume signals are received from satellites NS_2 and NS_3. So we have these three equations:

$$2\pi \frac{Da_1 - Db_1}{\lambda} = \mu_{x1}\cos\psi_x + \mu_{y1}\cos\psi_y + \mu_{z1}\cos\psi_z$$

(4) $$2\pi \frac{Da_2 - Db_2}{\lambda} = \mu_{x2}\cos\psi_x + \mu_{y2}\cos\psi_y + \mu_{z2}\cos\psi_z$$

$$2\pi \frac{Da_3 - Db_3}{\lambda} = \mu_{x3}\cos\psi_x + \mu_{y3}\cos\psi_y + \mu_{z3}\cos\psi_z$$

The object attitude is determined by solving equation (4) in terms of $\cos\psi_x$, $\cos\psi_y$ and $\cos\psi_z$. Most of the time the equation will be simplified using the following equation:

(5) $$\cos\psi_x^2 + \cos\psi_y^2 + \cos\psi_z^2 = 1$$

3-2- Differential carrier phase measurements for attitude determination

The difference between received signals from two antennas (separated from each other by a base line) is very important for attitude determination. Angle of wave front and wavelength is used to obtain the phase difference. (Fig. (2)). According to Figure (2) the phase difference is:

(1) $$be\cos\theta = \lambda(\Delta\phi - n)$$

Where be is the baseline length, θ is the angle between baselines and line of sight of GPS satellite, n is the integer number of wavelengths between two antennas, $\Delta\phi$ is the phase difference and λ is the wavelength of GPS satellite signal [3].

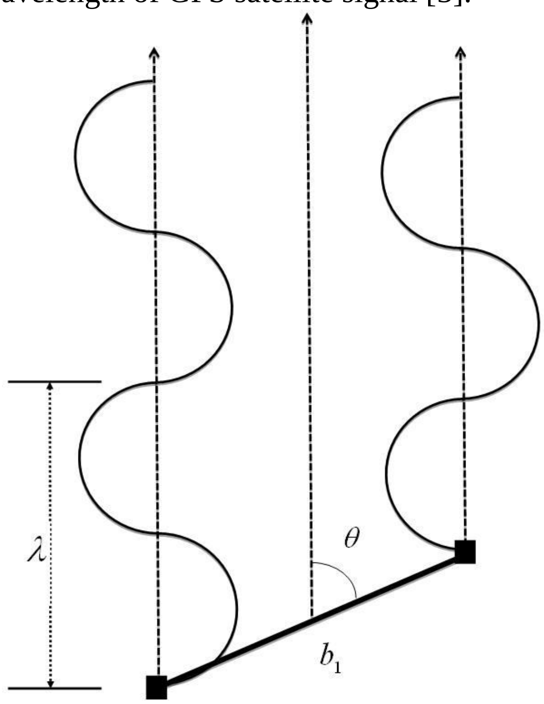

Fig. (2) : angle of wave front and wavelength to obtain the phase difference.

There are two carrier frequencies for GPS. One is in L1 band in 1575.42 MHZ and the other is on L2 band in 1227.6 MHZ. L1 is usually used in civil applications.

Assuming the known offset (integer number n) and compensated effect is the measured phase difference ($\Delta\phi$) is:

(2) $$\Delta\phi = \underline{b}^T A \underline{s} + v$$

Where ν is a zero mean white Gaussian process. The standard deviation assumed as σ^2 [3]. $S \in R^3$ is the normal sight line vector for the GPS satellite in the inertial coordinate system. $b \in R^3$ is also the normal baseline vector which shows the relative position vector from one receiver to the other one. The attitude matrix A is obtained from unit vertical matrix so that:

(3) $$\det A = 1 \quad , A^T A = I_{3\times 3}$$

3-3- Obtaining Wahba Cost Function for attitude determination

The most common methods for attitude determination are the ones in which Wahba problem is solved (this problem initially suggested by Wahba). Suppose m baselines and n visible GPS satellites. For *i*th baseline and *j*th satellite:

(9) $$\Delta\phi_{ij}\, \underline{b}_i^T A \underline{s} + \nu_{ij}$$

When: (10) $$V_{ij} \sim N(0, \sigma_{ij}^2)$$

So: (11) $$\Delta\phi_{ij} \sim N(\underline{b}_i^T A \underline{S}_j, \sigma_{ij}^2)$$

Consequently: (12) $$\rho_{\Delta\phi_{ij}}(\Delta\phi_{ij}) = \frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \exp\left[-\frac{\left(\Delta\phi_{ij} - \underline{b}_i^T A \underline{S}_j\right)^2}{2\sigma_{ij}^2} \right]$$

Assuming independence of measurements errors:

$$P_{\Delta\phi_{11},\dots,\Delta\phi_{mn}}(\Delta\phi_{11},\dots,\Delta\phi_{mn}) = P_{\Delta\phi_{11}}(\Delta\phi_{11}) \dots P_{\Delta\phi_{mn}}(\Delta\phi_{mn}) = \prod_{i=1}^{i=m}\prod_{j=1}^{j=n} P_{\Delta\phi_{ij}}(\Delta\phi_{ij}) =$$

$$= \prod_{i=1}^{i=m}\prod_{j=1}^{j=n} \left(\frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \right) \exp\left[-\frac{(\Delta\phi_{ij} - \underline{b}_i^T A \underline{S}_j)^2}{2\sigma_{ij}^2} \right] \qquad (13)$$

Now assuming the observed parameters as $\Delta\phi_{ij}{}'$, the problem is to find the matrix "A" so that the term $P_{\Delta\phi_{11},\dots,\Delta\phi_{mn}}(\Delta\phi_{11}{}', \dots, \Delta\phi_{mn}{}')$ be maximized with the Maximum likelihood criteria. Maximizing equation (13) is equal to maximize its natural logarithm. So:

(14) $$LnP_{\Delta\phi_{ij}}(\Delta\phi'_{ij}) = \sum_{i=1}^{i=m}\sum_{j=1}^{j=n} L_n \left(\frac{1}{\sqrt{2\pi\sigma_{ij}^2}} \right) - \sum_{i=1}^{i=m}\sum_{j=1}^{j=n} \left[\frac{\left(\Delta\phi'_{ij} - \underline{b}_i^T A \underline{S}_j\right)^2}{2\sigma_{ij}^2} \right]$$

σ_{ij} is considered constant, hence the problem is minimizing the second term, presented below:

(15) $$J(A) = \frac{1}{2}\sum_{i=1}^{i=m}\sum_{j=1}^{j=n} \left[\sigma_{ij}^{-2} \left(\Delta\phi'_{ij} - \underline{b}_i^T A \underline{S}_j\right)^2 \right]$$

As we see in the equation (15), σ_{ij}^{-2} is the same weight coefficient in Wahba problem.

3-4- How to solve Wahba problem for attitude determination

The problem is to find a vertical matrix minimizing the equation (16):

(16) $$J(A) = \frac{1}{2}\sum_{i=1}^{i=m} w_i \left\| \underline{b}_i - A\underline{s}_i \right\|^2$$

Where b_i is the *i*th unit vector of EFOV, b_i and s_i are known vectors. But the weight coefficients w_i should be determined. "A" is indeed the matrix of the direction cosines which should be determined. If the measurements are equally important, then w_i coefficients will be equal. Assuming $w_i = 1$ simplifies the problem.

A simple answer for attitude determination matrix in equation (17) using S-V-D (singular Value decomposition) method is:

(17) $$F = \sum_{i=1}^{i=n} w_i \underline{b}_i \underline{s}_i^T = U \sum V^T$$

The optimum answer for matrix "A" is:

(18) $$A_{opt} = U_+ V_+^T$$

U_+ , V_+ are obtained from the following equations:

(19) $$U_+ = U\left[diag(1,1,\det U)\right]$$

(20) $$V_+ = V\left[diag(1,1,\det V)\right]$$

Assuming $\delta_{\underline{\alpha}}$ is error vector of the inside angle, the error covariance is obtained as:

(21) $$\underline{P} = E\left\{\underline{\delta\alpha}\delta\alpha^T\right\} = \left[\sum_{i=1}^{i=m}\left(\sigma_{bi}^2 + \sigma_{si}^2\right)^{-1}\right]^{-1}\left(1 - FA_{opt}^T\right)^{-1}$$

When σ_{bi} , σ_{si} are standard deviations of measurement error process and sight axis in sequence. Position of the GPS satellites are known exactly, therefore we can assume $\sigma_{bi} >> \sigma_{si}$ and so $\sigma_i^2 \equiv \sigma_{bi}^2$.

3-5- Attitude determination Considerations

Common attitude determination systems use interferometric methods which have some physical limitations such as, Multipath Error, Line Bias Error, Antenna movements because of environmental turbulences (i.e. thermal distortion), Combination of accessible satellites, troposphere scattering and Crosswalk Errors.

Multipath error is the main error source and also the most complicated to solve. Despite progresses in modeling and reduction, this error is still the main source of error in evaluation of signal phases. Line bias errors which are due to Power downfall in RF cable, can be eliminated by calibration on the ground. The other source of error is the baseline slip. Generally longer baselines will cause more precision for attitude determination. But it is necessary to install the antennas on the flexible surfaces (such as solar arrays) to remove them easily. In these conditions, attitude determination is related to the point, antenna is mounted.

In the old methods and traditional applications, it was desired to use antenna with the most radio foresight. But in new methods multiple antennas are used so that every antenna has a portion of the space in its radio foresight. Therefore using multiple antennas causes the coverage of whole the space. By determining the satellite in foresight of every antenna and knowing Foresight vector of the antenna, matrix "A" can be obtained so that the GPS cost function (Wahba problem) become minimized.

4. Experimental applications of attitude determination using GPS

In attitude determination using GPS, it is expected to have a precision less than 1 degree in each direction, but it is not still reliable because of error in magnetic field model.
Anyway the basic idea on measuring the phase difference between GPS receiver signals to determine the attitude in three axes has been absolutely successful as it is tested on some satellites. One of first applications of this method was “RADCAL” satellite. In this satellite GPS receivers were used for attitude determination. This measurement was post processed.
In order to have the maximum foresight for GPS receiver and reducing the internal signals (they cause multipath repercussion) 3 patch GPS antenna installed on 3 sides of the satellite. Although baseline of the antenna is shorten, but the precision of attitude determination was about 2 degrees for each axis. The distance between antennas was 0.67 meters.
The other experiment was on Crista-SPAS satellite. It augmented the first real-time attitude determination. In this experiment satellite consists of a precise gyroscopic origin, but justification of orbit coordination toward reference coordination was not measured. This means small differences between 2 coordination systems may intrude in the calculations as small differences. Then both systems were measured. During the experiment the alignment tolerance between two coordination systems was about 2 degrees.
The other experimental case used GPS receiver for attitude determination, was UoSAT-12. This satellite was the first small satellite used 3-axis attitude determination, with a small budget. It also had the 3-axis stabilization. UoSAT-12 was launched to orbit in April 2000 (altitude 650 km, inclination of 64.5 deg). This satellite was equipped with many sensors for attitude determination. A multichannel GPS receiver was also used for orbit determination and precise time synchronization. This receiver was able to determine the attitude using an array of antennas (consist of 5 patch antennas). “MICROLAB”, ”GANE”, “OAST-Flyer” and “ORBCOMM” are some of the satellites used GPS receivers for attitude determination [3].

4-1- Comparison between attitude determination methods

Precision of GPS based Attitude determination is better than 0.1 degrees. Considering Table (1) and comparing the methods, we can say attitude determination using GPS is an acceptable method in precision and even better in some cases. In this method, attitude can be assessed in all 3 axes (Roll, pitch and Yaw) because phase difference of GPS signals is used. This is an advantage over other methods which can only give the attitude in one or two axes.
This method is also more economic because it uses the same GPS for navigation and time synchronization of the satellite. Indeed a GPS receiver in a satellite can be multipurpose and eliminate the need to other instruments and so reducing the cost. The other advantages are high reliability and ease of use compared to other methods.

5. Conclusions

Based on theories described, a GPS can be used to determine the attitude of flying objects and specially satellites. It can be independent or supplementary for other methods. Attitude determination using GPS is due to phase difference in received signals by antennas. At least 2 receivers and 4 signal receiving antenna is necessary. The accessible precision would be about 0.01 degree.

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