

## A Novel Approach of Classification in Simulation Optimization Methods



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### Abstract

Showing smooth mathematical formulations in optimization for many real world problems would be too complex. In such situations, we use simulation to discover at least one admissible solution. Simulation Optimization (SO) has wide usages in this case with the aim of managing the intricate stochastic complex systems whose performance can only be evaluated by simulation. Simulation Optimization offers a structured approach to stochastic complex system design when analytical expressions for input/output relationships are unavailable. “Methods” domain is one of the six domains defined in Simulation Optimization. Therefore, modifying the practical Methods in deterministic optimization to coordinate the stochastic environment in Simulation Optimization has prominent role. In fact, in this paper, we present all Methods one can employ in Simulation Optimization.

**Key words:** Simulation Optimization, Stochastic Optimization, Response Surface Methodology, Stochastic Approximation

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### 1. Introduction

In the past, simulation and optimization technologies were separated in practice, whereas a large body of research in this environment was disclosed the significance of combining them. Recently, Simulation Optimization has become an area which attracts the researchers' attention.

Many decision making problems determine some decision variables that generate the most favorable value for some performance criteria called them responses in design of experiments. Inventory and queuing problems are two generic problems with involvement of random conditions that are uncontrollable in the operation research literature. In fact, what makes Simulation Optimization different from the ordinary deterministic

optimization setting is its stochastic nature. For simple problems, analytical techniques can be applied (Ross, 2003). Those analytical techniques become inapplicable when the problem gets more complicated. Instead, computer simulation model has been demonstrated to be a powerful tool (Kao & Chen, 2006). In the context of Simulation Optimization, a computer simulation model can be applied as a mechanism that turns input parameters into output performance measures (Law & Kelton, 1991). Simulation Optimization can be defined as the process of finding the best input values from among all possibilities without explicitly evaluating each possibility (Carson & Maria, 1997).

Nowadays, the Simulation Optimization technology is to overcome the limitation of computer simulation methodology. Computer simulation methodology has been applied by a complex system analysis more widely. But, it could not provide more helpful decision making function. Combining the simulation mechanism and the optimization strategy, the Simulation Optimization technology could not only aid or improve decision making of the simulation, but also build the stochastic complex system model easily that is quite difficult to be represented by traditional optimization methods. As a result, Simulation Optimization offers a structured approach to stochastic complex system design when analytical expressions for input/output relationships are unavailable (Rosen et al., 2007; Guo et al., 2006).

Traditional Simulation Optimization problem is defined in Eq. 1 in the case of a single objective (Fu, 1994; Fu, 2002):

$$\begin{array}{ll} \text{Minimize (Maximize)} & f(X) = E[F(X, W)] \\ \text{Subject to.} & X \in \mathbb{X}, \end{array} \quad (1)$$

where  $X \in \mathbb{X}$  denotes the vector of input variables.  $f(X)$  is the objective function or is the unknown expected value of the system performance measure estimated by  $\hat{f}(X)$ . This is obtained from sample performances of a simulation model  $F(X, W)$  observed under feasible input parameter  $X$ , ( $X$  divided into two categories of discrete or continuous).  $W$  represents the stochastic effects of the system resulting in a specific sample path (simulation replications). The constraint set  $\mathbb{X}$  could be either explicitly given or implicitly defined.

Alternative way of formulating the general Simulation Optimization problem was proposed by Azadivar (1992):

$$\begin{array}{ll} \text{Maximize (Minimize)} & f(X) = E[F(X, W)] \\ \text{Subject to.} & P\{(z(X) = E[Z(X, W)]) \leq 0\} > 1 - \alpha \\ \text{And} & u(X) \leq 0 \end{array} \quad (2)$$

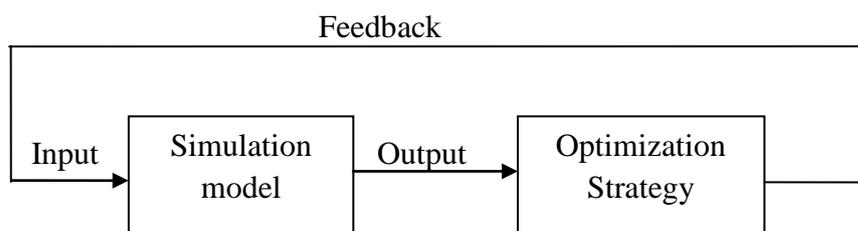
where  $\alpha$  is the vector of risks of violation of constraints which is offered to the decision maker to accept.  $z(X)$  is described in a similar fashion as  $f(X)$  (in Eq. 1) to be the unknown expected value of the system performance measure which is obtained from sample performance measure of a simulation model on  $Z(X, W)$ .  $u(X)$  is a vector of deterministic constraints on the decision variables.

Simulation Optimization problems could be viewed as optimization problems of a system whose outputs (performances) can only be evaluated by simulations (Fu et al., 2005). Thus the objective of Simulation Optimization is to find the optimal settings of the input

variables to the simulated system that makes the output variables at their best or optimal conditions (Horng & Lin, 2009).

Most of the Methods in this case are robust with respect to the number of replications to estimate  $f(X)$  for each  $X$  conducted at each run and have a major determinant of the computational cost for a Simulation Optimization algorithm (Fu, 2002).

A Simulation Optimization model is shown in Fig. 1. The outputs from the simulation model which evaluate the outcomes of the inputs fed into the model are utilized by the optimization strategy. The optimization strategy determines a new set of input values on the basis of this evaluation and other evaluations with these new outputs of simulation model (Glover et al., 1999).



**Fig 1:** Simulation Optimization Model.

It is important to mention that the Simulation Optimization techniques based on uncertainty, risk and multiple responses environment can obtain higher quality final solutions compared with similar techniques founded on deterministic decision making strategies in real world.

This paper is organized as follows. Domains of Simulation Optimization are introduced in section 2. Section 3 describes the Methods and classification. Section 4 condenses the applications of Simulation Optimization in real business and industry case. Finally, a summary of existing studies and a discussion on the future research are discussed.

## 2. Domains of Simulation Optimization

The work of Dennis E. Smith (1973a, 1973b) was extended by Bowden and Hall (1998) by representing six distinct domains. Domains of Simulation Optimization are: Interfaces, Problem Formulation, Classification, Strategy and Tactics, Intelligence, and “Methods”.

This paper focuses only on one of these six domains, namely Methods domain. However, the details of other domains could be found in Boesel et al. (2001). The Methods domain addresses those optimization methods used to optimize simulation systems.

Lately, new Methods for Simulation Optimization technology have been developed based on a principle which separates the simulation model from different optimization algorithms. The details can be found in several review articles (see, Fu, 1994; Fu, 2002; Azadivar, 1992; Fu et al., 2005; Meketon, 1987; Jacobson & Schruben, 1989; Safizadeh, 1990; Azadivar, 1999).

One of the most important points to solve Simulation Optimization problems is to choose the correct way of solving their problems. This paper presents comprehensive investigation of all the Simulation Optimization Methods. With regard to the division and

the separation of all the Methods that have been done, solving any problem in Simulation Optimization environment is clear. These Methods find a direction of promotion, and move a distance from the current point along that direction. This process is repeated until no further movement can be made.

### 3. The Methods & Classification in Simulation Optimization

Existing Methods can be classified into Local optimization and Global optimization. Local optimization Methods (as a special case of optimal control problems in our classification scheme) are further broken down into two distinct categories based on the nature of decision space (Fu, 1994). These two categories are discrete or continuous variables. On the other hand, we can say that the categories of inputs in Local optimization Methods are generally divided into two types: qualitative and quantitative. The later are then divided into two distinct domains of discrete and continuous variables (Fu, 2002). The discrete case could be further subdivided into Finite and Infinite parameter space. Local optimization techniques differ from Global optimization techniques in the assumption about the shape of the response surface value. Local optimization techniques are not useful for the simulation models that are complex and multimodal. They are often stuck in a local optimum and develop poor solutions if there are no effective methods to find good initial solutions. Global optimization techniques are the heuristic methods such as Genetic Algorithms and Tabu Search. Heuristic methods are developed to help a search escape from the local optimum and submit better solutions (Fu, 1994; Andradottir, 1998; Tekin & Sabuncuoglu, 2004).

Occasionally, we could divide the discrete case into the domains of Ordered and Unordered instead of Finite and Infinite parameter space. This subdivision includes two conditions of quantitative and qualitative decisions (Fu, 1994). In this paper, discrete variables are broken down into Finite and Infinite parameter space.

Simulation Optimization problems can be classified in the several ways. Each class can be considered as a special case in the equation of Simulation Optimization. The problem is reduced to a single objective optimization, if  $f(X)$  in Eq. 1 is a one-dimensional vector while in its general form it is a multiple objective problem. If variables of  $X$  are continuous the problem is often easier to solve by available stochastic search Methods. If they are discrete but still quantitative, problem will be closer to integer programming techniques. If  $X$  renders a vector of qualitative decision policies, optimization will become more difficult because of the lack of available analytical tools to behave this type of problems (Carson & Maria, 1997; Azadivar, 1999). Many researchers have investigated the Simulation Optimization problem as a single objective optimization. But, simulation models are dealing with systems having multiple and conflicting performance measures, which cannot be represented by a single function, whereas this condition have been occurred in the real world.

The distinction is important because the tools that are presently used to attack the three categories of problems are quite different. Choosing the type of Simulation Optimization Methods to solve the Eq. 1 of this type of problems is dependent on whether the system of interest consists of continuous parameters or discrete parameters or wants to use heuristic Methods.

Approaches for heuristic Methods of Simulation Optimization include:

- 1) Nelder-Mead Simplex Search Method, the fond researchers can see in the papers such as (Kao & Chen, 2006; Carson & Maria, 1997; Rosen et al., 2007; Barton & Ivey, 1996).
- 2) Genetic Algorithm (GA), (Carson & Maria, 1997; Rosen et al., 2007; Horng & Lin, 2009).
- 3) Evolutionary Algorithms (EAs), (Carson & Maria, 1997; Yu et al., 2010).
- 4) Simulate Annealing (SA), (Carson & Maria, 1997; Rosen et al., 2007; Azadivar, 1992; Horng & Lin, 2009; Azadivar, 1999).
- 5) Tabu Search (TS), (Carson & Maria, 1997; Horng & Lin, 2009; Glover et al., 1999).
- 6) Scatter Search (SS), (Fu, 2002; Glover et al., 1999).
- 7) Particle Swarm Optimization (PSO), (Kuo & Yang, 2011).

Proposing Methods for continuous parameters are:

- 1) Response Surface Methodology (RSM), (Carson & Maria, 1997; Rosen et al., 2007; Fu, 2002; Fu et al., 2005; Azadivar, 1999; Biles, 1974; Smith, 1976; Daugherty & Turnquist, 1980; Wilson, 1987; April et al., 2003).
- 2) Stochastic Optimization divide to Stochastic Approximation Methods and Sample Path Optimization Methods, (Kao & Chen, 2006; Rosen et al, 2007).
- 3) A-Teams is a process that involves combining conventional algorithms such as Newton's Method and Levenberg-Marquardt Algorithm, (Kao & Chen, 2006).
- 4) Gradient Based Search Methods are broken to Finite Difference Estimation, Likelihood Ratio Estimators, Perturbation Analysis and Frequency Domain Experiments, (Carson & Maria, 1997; Rosen et al., 2007; Azadivar, 1999).

Methods for discrete parameters of Finite parameter space are:

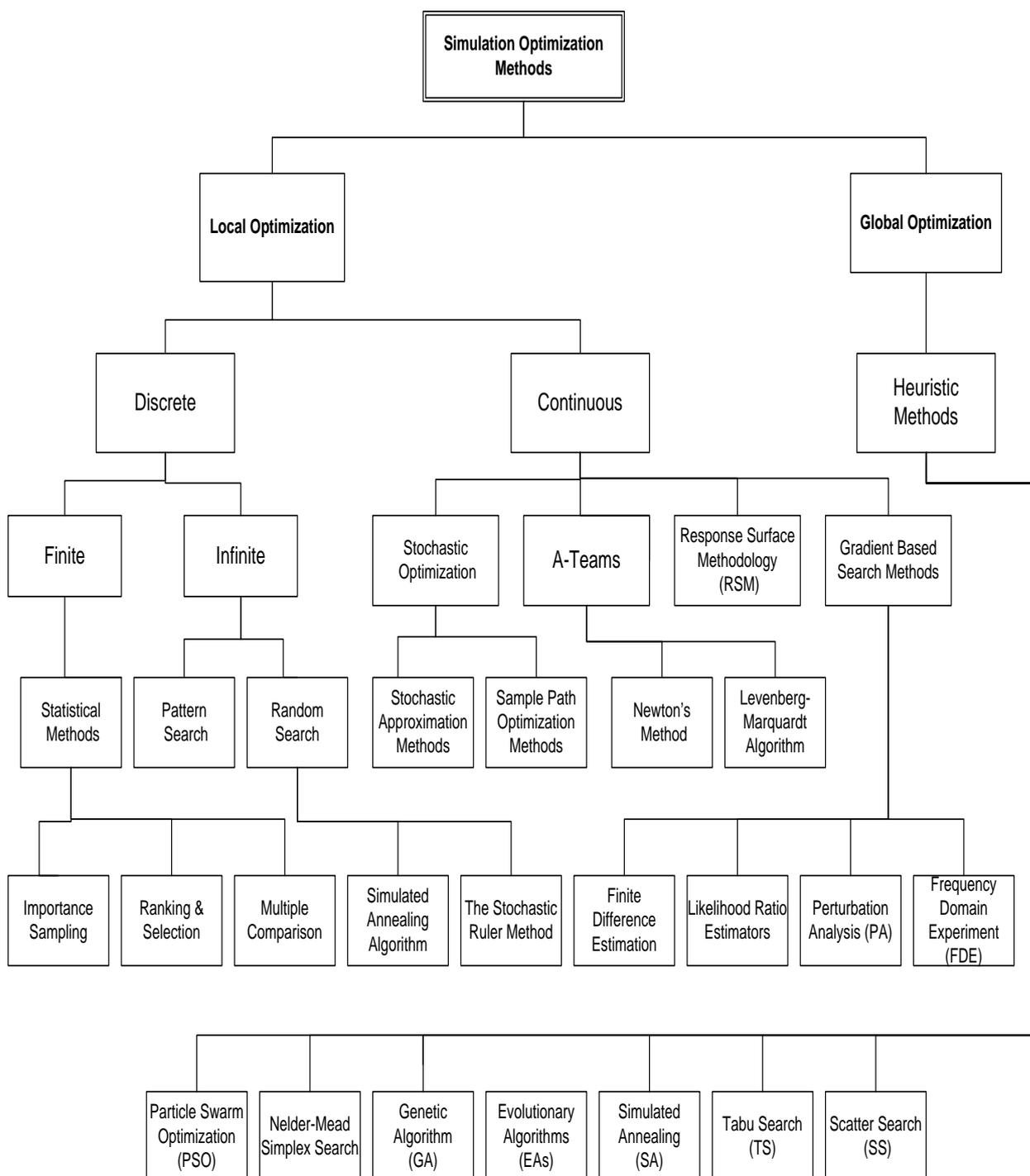
- 1) Statistical Methods that subdivided to Importance sampling, Ranking and selection and Multiple Comparison when problems consist of a small solution space are used, (Fu, 2002; Fu et al., 2005; April et al., 2003).

On the other hand, the Methods for Infinite parameter space are: 2) Random Search and 3) Pattern Search, (Rosen et al., 2007; Andradottir, 1998). The classification of methods, done in this research, is shown in Fig. 2.

Angun and Kleijnen (2005) divided Simulation Optimization Methods into two categories of White-Box and Black-Box Methods. The White-Box Methods are estimated the gradient from a single simulation run. On the other hand, the Black-Box Methods are implied to the methods which treat the simulation optimization problem as a Black-box. In this case no gradient information is available. On the basic of this classification, this paper also tries to consecrate the Methods of Simulation Optimization technology in each class, too. Chart of Fig. 3 is illustrated this type of classification.

In addition, I believe the prospects for the so-called Ordinal Optimization (OO) approach of Ho et al (Ho et al., 1992; Ho et al., 2000) have been fully exploited in Simulation Optimization. The basic idea of this approach is that it is much easier to determine

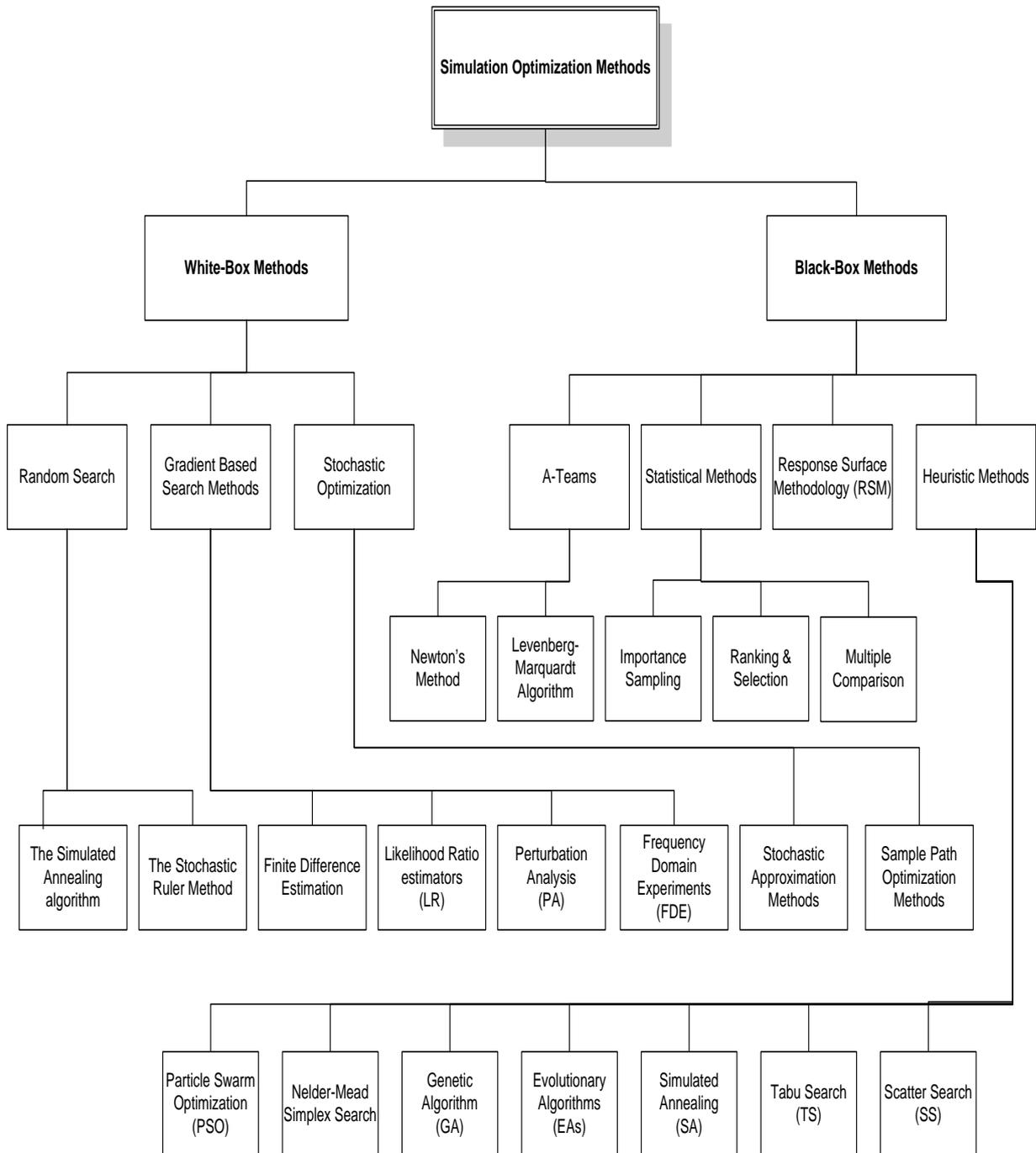
approximate order than precise estimation. In fact, to cope with the computational complexity of this problem, we will employ the Ordinal Optimization which seeks a good enough solution with high probability instead of searching the best solutions (Ho, 1999). So, we can avoid one of the barriers in Simulation Optimization. This barrier is cost of replications. An Ordinal Optimization Method was rendered a good enough solution of the



**Fig 2:** Classification of SO methods on the basis of global and local optimization.

stochastic solution optimization problem with huge input variable space (Fu, 1994; Horng & Lin, 2009).

The main future idea of this work exists in the resolution of a real industrial problem, having integrated the RSM optimization in a multi objective model, together with the optimization with simulation of stochastic model. Nowadays, RSM is widely used for



**Fig 3:** Classification of SO methods on the basic of White-Box and Black-Box methods.

Simulation Optimization. Particularly, RSM is appropriate procedure for solving complex systems where little prior knowledge is available. So, in mentioned Methods we are going to improve and expand RSM in further research. We believe that with automation of mentioned Methods especially in RSM framework to perform these problems, we can make this task more feasible and applicable. Also we recommend expanding the current framework to solve constrained problems and accomplish additional effort on advancing concepts such as multi-responses, uncertainty and risk in it. Researchers can make Simulation Optimization procedures more suitable to be interfaced with real world conditions.

#### **4. The Applications of Simulation Optimization**

Applications of Simulation Optimization technology are various. They respond a broad surface of business activities. The major application areas are manufacturing, process design, industrial experimentation, design optimization, communications networks, computer, reliable optimization and business processes (Fu, 2002). To illustrate, the user of simulation and other business or industry evaluation models may want to know the goals of Simulation Optimization. we can mention some of them such as: The best factory layout, The best investment portfolio, The safest equipment replacement policy, The most cost effective inventory policy, The best workforce allocation, The most productive operating schedule, The best layouts of machines for production scheduling, The best supply chain management, The best human resource planning, The best location, The best set of treatment policies in waste management, and many other objectives that have been done (Fu et al., 2005).

Simulation Optimization Methods were used to study economic issues related to multi period asset allocation problems in practical settings (Yu et al., 2010). Heuristic Methods of Simulation Optimization namely PSO was proved that it is better than other Methods for solving assembly line design problem (Kuo & Yang, 2011). Simulation Optimization Methods like A-Teams can be used for optimizing a Kanban sizing problem (Hall & Bowden, 1996).

#### **5. Conclusion**

The choice of the appropriate Method in Simulation Optimization problems depends on the problem that analyst wants to be solved. In summary, optimization has been one of the most exciting regions in simulation regions because it improves the utility of simulation modeling. This paper discusses the “Methods” for using simulation as a tool for optimization of stochastic complex systems that are modeled by computer simulation and classifies these Methods for straight selection of the best Method with regard to the qualification of the Simulation Optimization problem. The main contribution of this work exists in the resolution of a real industrial problem, having introduced all of the Simulation Optimization Methods, together with the classification of Methods in optimization with simulation of stochastic model. Different researches classified the Simulation Optimization only into global and local optimization or into Black-Box and White-Box Methods without any details. One of the main contributions of this paper is the complete classification of the current methods related to simulation optimization, shown in Fig. 2 and Fig. 3. The advantage of these figures is we can find the best method quickly. Making

complex decisions in the environment of uncertainty, risk and multi responses would be useful for users of Simulation Optimization. There are a number of aspects of these Methods and domains of Simulation Optimization which bear further scrutiny.

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