Investigation of Free-Convection Heat Transfer for an Al-Nanofluid Solution Model

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Abstract

The aim of this paper is investigation of heat transfer for a nanofluid system and the prediction of Nusselt number was done. The supposed Al-water model, Al as nanoparticle and water as base fluid, was assumed to locate beside a vertical flat wall. The investigations were done for different amounts of volumetric fractions of nanoparticles, Grashof numbers and different models of viscosity. A total integral method was utilized for calculations. The results show that, Nusselt number has a direct dependency on viscosity, for which, the optimum model of viscosity is presented for the heat transfer improvement. All graphs are depicted for Nusselt changes versus the other parameters. The obtained results have very good agreement with the experimental data.

Keywords: Al nanoparticle, nanofluid, free-convection

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1. Introduction

Nowadays, the global demand of capable, reliable, and economic heat exchanging systems is noticeable for all industries. The possibility of energy saving and environmental pollution decrease will become doable by applying proper principles and designs. Generally, raising the heat transfer rate may be done by two methods of passive and active. The first category doesn`t need any external force, while the latter requires an external force or power. Utilizing additives in liquids is involved in the passive method. The application of solid particles in liquid was started around 100 years ago, and the particle sizes were milimeteric or micrometric, Penny (1956). According to the problems, made by large particles, researchers made many efforts to use particles with smaller size in nanometeric scales. Adding nanoparticles including metallic, nonmetallic, composite, and metallic oxide particles into fluid can be resulted in preparing nanofluids. Nanofluids are made of two phases, solid phase including nanoparticles and liquid phase including a base fluid. Since the particles have homogeneous distribution in the fluid, nanofluids can be considered as mono-phase. Nanofluids cause heat transfer and fluid motion properties improvement, and lead to less pressure loss. Also, carbon nanotubes can cause electrical properties and heat conduction improvement, for which, these are highly considered by industries, Petal (2003) ,Orozeco(2005), Barkhodari(2005). According to the research works, adding nanoparticles can make better heat transfer rate and the heat transfer increase depends on particle type and size, type of base fluid, pH, and concentration, Maxwell (1873), Incropera(1996) , KEric (1990). In aspect of pH, the best conditions of nanofluid heat transfer are located in lower pH, Drew (1937). Nanofluids influence all heat transfer mechanisms, especially free and forced convection. Most of research about forced convection of nanofluids has been experimentally, and less theoretical works have been done. According to the experimental results, forced convection coefficient (h) and Nusselt number are raised by adding nanoparticles onto base fluid and beside all the mentioned factors, Nusselt increase is dependent on Reynolds number, Das (2003) , Estman (2001) , Putnam (2006). Free convection heat transfers may be analyzed by both experimental and theoretical methods, but there are fewer models which can explain Nusselt number with lower error percentage. In this study, free heat transfer coefficient and Nusselt number changes are simulated, using an integral solution by assuming a physical model for a vertical wall (Fig. 1). The modeling is presented for both UHF and UWT boundary conditions and the assumed properties are shown in table 1. Considering other research works, an optimum viscosity model is considered to decrease the calculation errors. About the nanoparticle and base fluid of this study, some interesting experimental researches have been done, Foutukian (2010). Our calculations show proper agreement with

Property	Fluid phase (water)	Solid phase (Al_2O_3)

other experimental works, Xuan (2005). Nusselt numbers are investigated for the particle volumetric fractions of less than 5%, which are the percentage limitations that have been investigated by other researchers. Since the particles are assumed to be uniform and spherical, the obtained model is not applicable for carbon nanotubes.

$Cp\ [J\ kg-1K-1]$	4179	7656
$k\ [W\ m-1\ K-1]$	0.613	40
β [K^{-1}]	21×10^{-5}	2.4×10^{-5}
ρ [kg m^{-3}]	997.1	3970

Nomenclature

Pr	Prandtl number
$Cp\ [J\ kg-1K-1]$	heat capacity
$k\ [W\ m-1\ K-1]$	thermal conductivity
UHF	uniform heat flux
UWT	uniform wall temperature
Gr	Grashof Number
Gr^{*}	modified Grashof number
Nu	Nusselt number
g [m s^{-2}]	gravitational acceleration
h[W m^{-2} k^{-1}]	local heat transfer coefficient
Ra	Rayleigh number
T [K]	dimensional temperature
U [m s^{-1}]	dimensional x
Greek symbols	
μ [N s m^{-2}]	dynamic viscosity
ν[m^2s^{-1}]	kinematics viscosity
δ_t	thermal boundary layer thicknesses
δ	dynamical boundary layer thicknesses
φ	volume fraction
β [K^{-1}]	volumetric expansion coefficient
ε	the heat transfer performance
α	constant
Δ	ratio of the thermal boundary layer thickness *(δ_T)* to that of the dynamical(δ)
ρ [kg m^{-3}]	density
θ	dimensionless temperature
Subscripts	
p	nanoparticle
bf	base fluid
r	ratio of the nanofluid (nf) to that base fluid (bf)

Table 1. Physical properties of liquid- solid phase used in present study.

2. Physical model

Considering all previous research works, we attempted to select a new model and two new boundary conditions for Nusselt estimation. In this model, we assume that the temperature gradient is neglected along boundary layer, otherwise the temperature changes cause changing in nanoparticle properties and flocculation which may lead to micrometric size and operational problems, Zeinali Heris (2006) ,Wen(2003). The nanofluid is taken as Newtonian fluid and the particle sizes are in the range of 32-60 nm. For purpose of high accuracy, the temperature and velocity profiles are assumed to be power-four polynomial.

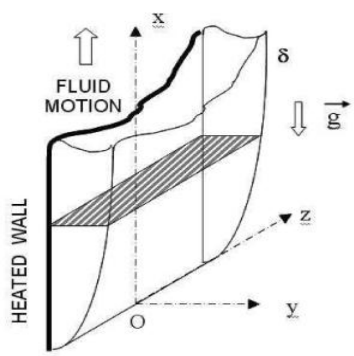

Fig 1: The physical model

$$\begin{cases} U = \dfrac{g\beta\rho_{\omega}\Omega\partial^{3}}{12\lambda v}\left[-\eta^{4}+3\eta^{3}-3\eta^{2}+\eta\right] & (1) \\ \theta = (T-T_{\infty}) = \dfrac{\rho_{W}\Omega\partial}{2\lambda}\left(-{\eta_{T}}^{4}+2{\eta_{T}}^{3}-2\eta_{T}+1\right) & (2) \end{cases}$$

Where, $\eta = {}^{y}/_{\sigma} \leq 1$ and $\eta_{T} = y \,/\, \sigma_{T} \leq 1$, λ indicates fluid conductivity, and φ_{ω} is heat flux density.

By substituting temperature and velocity terms into the following integrals, and simultaneous solution by MATLAB software,

$$\frac{\partial}{\partial t}\int_{0}^{\delta} U dy + \frac{\partial}{\partial x}\int_{0}^{\delta_{nf}} U^{2}\, dy = g\beta_{nf}\int_{0}^{(\Omega\delta)_{nf}} (T-T_{\infty})\, dy - \vartheta_{nf}\left(\frac{\partial U}{\partial y}\right)_{y=0} = 0 \qquad (3)$$

$$\frac{\partial}{\partial t}\int_{0}^{\Omega\delta} \theta dy + \frac{\partial}{\partial x}\int_{0}^{(\Delta\delta)_{nf}} (T-T_{\infty}) U dy = \frac{\square_{nf}}{Pr_{nf}}\left(\frac{\partial T}{\partial y}\right)_{y} = 0 \qquad (4)$$

Δ is obtained as:

$$\Delta = 1.576\times10^{-4}K^{4} - 4.227\times10^{-3}K^{3} + 4.282\times10^{-2}K^{2} - 0.1961K + 0.901 \qquad (5)$$

Where, k= $lnPr$

Now, the calculations of Nu are presented. According to the above results, considering the Furrier's law, and continuing the computations, the following relations are obtained for h:

$$h_{nf}(UWT) = \left[\frac{25g\beta_{nf}\theta_{\omega}{k_{nf}}^{4}}{378(9\Delta_{nf}-5){\Delta_{nf}}^{4}{\vartheta_{nf}}^{2}x}\right]^{\frac{1}{4}} \quad (6)$$

$$h_{nf}(UHF) = \left[\frac{2g\beta_{nf}\varphi_{\omega}{k_{nf}}^{4}}{27(9\Delta_{nf}-5){\Delta_{nf}}^{4}{\vartheta_{nf}}^{2}x}\right]^{\frac{1}{5}} \quad (7)$$

Considering the particles type and the heat transfer source, the average Nu may be obtained as

$$\overline{Nu_{nf}} = \frac{\overline{h_{nf}}L}{k_{bf}} \quad (8)$$

Here, by taking two boundary conditions of UHF and UWT, the Nusselt terms may be obtained as

$$\overline{\mathrm{Nu_{nf}}}(\mathrm{UWT}) = \frac{4\sqrt{5}}{3\Delta_{\mathrm{nf}}}\left[\frac{\beta_r k_r^4}{378\nu_r^2(9\Delta_{nf}-5)}Gr_{bf}\right]^{\frac{1}{4}} \quad (9)$$

$$\overline{\mathrm{Nu^*_{nf}}}(\mathrm{UHF}) = \frac{6}{5}\left[\frac{2\beta_r k_r^4}{27\nu_r^2(9\Delta_{nf}-5)\Delta_{nf}^4}Gr^*_{bf}\right]^{\frac{1}{5}} \quad (10)$$

For the UHF case, the modified Grashof number can be defined as

$${Gr_{bf}}^{*} = \frac{g\beta\rho_w L^4}{K_{bf}{\vartheta_{bf}}^{2}} \quad (11)$$

Model	Nu	
I	0.5163(0.4436+$\phi^{1.0809}$)$Gr^{0.3132}$	$Khanafer(2003)$
II	$0.148(1+\phi)^{-0561}Ra^{0.298}$	$CJ(2008)$
III	$0.124Ra^{0.318}(-2.41\phi+1)$	$Abouali(2009)$

Table 2 . The investigated theories

As mentioned in the previous part, the properties of base fluid vary by adding nanoparticles. These variations cause deriving new correlations for calculating the thermo-physical properties of nanofluids, which are presented as follows:

$$\rho_{nf} = (1-\varphi)\rho_{nf} + \varphi\rho_p \quad (12)$$

$$(C_p)_{nf} = (1-\varphi)(C_p)_{bf} + \varphi(C_p)_p \quad (13)$$

$$(\rho\beta_{nf}) = (1-\varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_p \quad (14)$$

There exist many terms for estimating nanofluid conductivity, Tsai (2004), ögüt (2009) ,Nada (2008). We tried to select a proper model and among all, the Wasp model, Lo (2005), Maxwell (1904) was chosen:

$$K_r = \frac{K_p + 2K_{bf} - 2\varphi(K_{bf} - K_P)}{K_p + 2K_{bf} + \varphi(K_{bf} - K_p)} \quad (15)$$

In the obtained relations, it`s obvious that the Nusselt number is dependent on viscosity via Pr. The following model is selected as a proper viscosity model:

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = (1 + 2.5\varphi) \qquad \text{for } \varphi < 0.05 \quad (16)$$

3. Results and Discussions

The calculations results for two boundary conditions of UHF and UWT for the vertical wall are depicted in figures 2 and 3. It`s obvious from the figures and the other research works that, the free convection Nusselt number is increased by raising the particles volumetric concentration. Also, the agreement the results with experimental researches show that the applied theory and method of this study would be competent, Ekert (1960) .

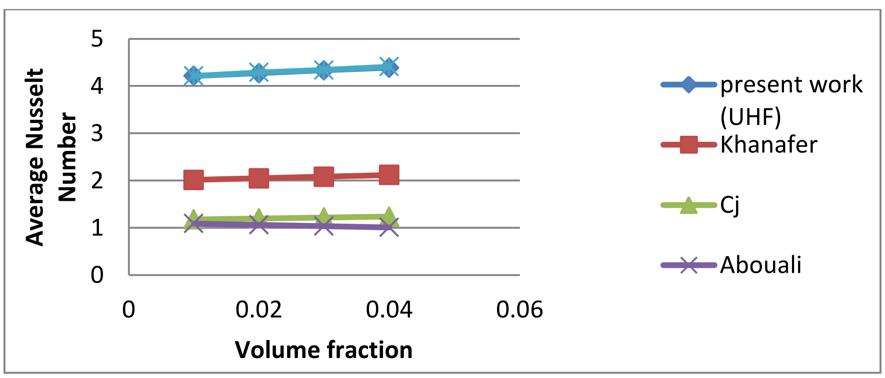

Figure 2: Comparing the results of UHF boundary conditions with the experimental results

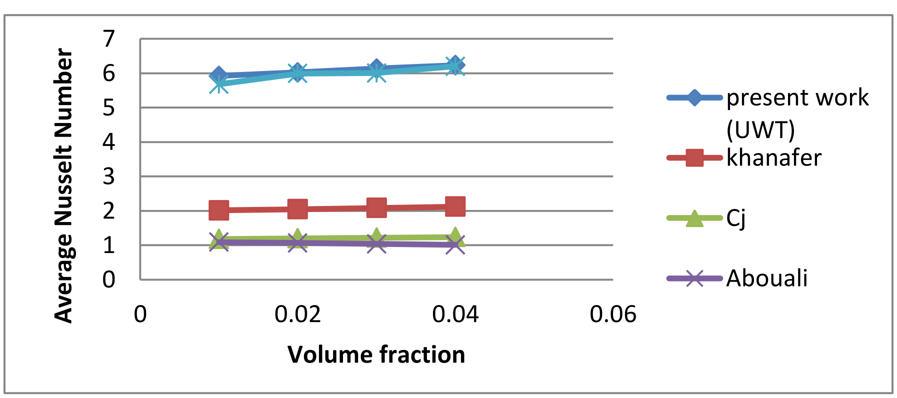

Figure 3: Comparing the results of UWT boundary conditions with the experimental results

According to Khanafer (2003), Nu is raised by increasing volume fraction as well. But the low amount of increase is because of considering the base fluid heat transfer coefficient in their calculations, while we chose k_{nf} in our model calculations, Kasaeian (2011). Regarding the difference between our results and CJ (2008), they have utilized different viscosity models without considering an optimum model and also the density change of the base fluid (water) is neglected. Also, Abouali (2009) etal. have considered an ideal state for the boundary conditions.

4. Conclusions

The calculations and graphs reveals the problems of some previous works and between two boundary conditions, UWT is more effective in aspect of increasing Nu. Among all the theories, the theory and model of Abouali (2009) may be approved by some modifications and changing boundary conditions.

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