Robust Passification of Uncertain Linear Systems with State Delay

Najmeh Eghbal*, Ali Taheri Anaraki, Behzad Ghayoumi Anaraki*****
*** Islamic Azad University, Sabzevar Branch, Iran, najmeh.eghbal@gmail.com**
**** ali.taheri.elc@gmail.com**
***** Islamic Azad University, Sabzevar Branch, Iran,**
behzad.ghayoumi.anaraki@gmail.com

Paper Reference Number: 1812-463
Name of the Presenter: Najmeh Eghbal

Abstract

This paper deals with the passification problem of a class of uncertain linear systems with time delay. The time delay is assumed to be unknown and the system under consideration is subjected to time-varying norm-bounded parameter uncertainties in the state matrices. Attention is focused on the design of a state-feedback controller which guarantees the passivity of the closed-loop system for all admissible uncertainties. The gain of this controller is the solution of some linear matrix inequalities (LMIs). The numerical example demonstrates the validity and applicability of present approach.

Key words: Linear Parameter Varying (LPV); passivity; parameter uncertainty.

1. Introduction

Passification is understood as finding a state or output feedback rendering the closed-loop system passive. Passive systems can not generate energy on their own. It can be shown that if the system is state strictly passive or dissipative, the origin is an asymptotically stable equilibrium point and the storage functions induced by dissipativity, usually provide natural candidates for Lyapunov functions. This means that the concept of passivity and energy consideration leads to stability [1]. Based on this introduction of the notion of passivity and dissipative systems, many researchers have considered the passivity-based analysis and synthesis techniques that are highly effective. It is well known that delays appear in dynamical systems; make the dynamical behavior of system more complicated than that of systems without any delays. In recent years, the passivity and passification of linear delayed systems were studied [2-4].

On the other hand, inaccuracy in mathematical modeling usually leads to uncertainties. Time delays and uncertainties have important impacts on the dynamic behavior of the systems.

However to our knowledge, little attention has been paid to the passification problem in the simultaneous present of time delay and parametric uncertainty [5-8]. This motivates the present research on robust passification for linear systems against unknown time-delay and admissible norm-bounded uncertainty which appear in all the matrices of the state space model and may be time-varying. This problem aims to designing a state-feedback controller such that for all admissible parameter uncertainties, the closed-loop system remains robustly passive, independent of the time-delay. The problem can be cast in a linear matrix inequalities (LMI) format which can be conveniently solved by semi-definite programming techniques [9]. Numerical example is included to illustrate the analytical development.
Notations: R^n denotes the n-finite-dimensional Euclidean space; $R^{n\times m}$ denotes the set of all $(n\times m)$-matrices; A^T and A^{-1}, respectively, denote the transpose and inverse of matrix A; and I denotes identity matrix of appropriate order. The notation $X \succ 0$ $(\prec)$ denotes a symmetric positive definite (negative definite) matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

The rest of this paper is organized as follows. In section II, the problem to be studied is stated and some preliminaries are presented and in section III main results are proposed. In section IV, numerical example is given to demonstrate the effectiveness of the theoretical results. And finally, conclusions are drawn in section V.

2.Problem Formulation and Preliminaries

Consider the following system

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A(t))\,x(t) + (A_1+\Delta A_1(t))x(t-\tau) + (B+\Delta B(t))\,w(t) \\ y &= (C+\Delta C(t))\,x(t) + (C_1+\Delta C_1(t))x(t-\tau) \end{aligned} \tag{1}$$

$x\in R^n$ is the state vector, $w\in R^m$ is the exogenous input, $y\in R^q$ is output vector and τ denotes the unknown delay. A, A_1, B, C, C_1 are real known constant matrices of appropriate dimensions, $\Delta A(t), \Delta A_1(t), \Delta B(t), \Delta C(t)$ and $\Delta C_1(t)$ are uncertainties which are assumed to be of the form

$$\begin{aligned} \Delta A(t) &= H_0 F_0(t) E_0 \\ \Delta A_1(t) &= H_1 F_1(t) E_1 \\ \Delta B(t) &= H_2 F_2(t) E_2 \\ \Delta C(t) &= H_3 F_3(t) E_3 \\ \Delta C_1(t) &= H_4 F_4(t) E_4 \end{aligned} \tag{2}$$

where H_i, E_i $i = 0,1,2,3,4$ are real known constant matrices of appropriate dimensions and $F_i(t), i = 0,1,\ldots,4$ are unknown real matrices that satisfy the following

$$F_i^T(t)F_i(t) \le I,\ i = 0,1,\ldots,4\,. \tag{3}$$

Definition 2.1: The system (1) is called passive if there exist a scalar $\gamma \ge 0$ such that

$$2\int_0^{t_p} y^T(t)w(t)dt \ge -\gamma \int_0^{t_p} w^T(t)w(t)dt \tag{4}$$

for all $t_p \ge 0$ and for all solutions of (1) with $x(0) = 0$ [1].

Lemma 2.1: [10] Let H,F and G be real matrices of appropriate dimension, then for any scalar $\varepsilon \succ 0$ and a matrix F satisfying $F^T F \leq I$, we have

$$HFG + G^T F^T H^T \leq \varepsilon HH^T + \varepsilon^{-1} G^T G \tag{5}$$

Lemma 2.2: (Schur complement theorem) The linear matrix inequality $\begin{bmatrix} H & S^T \\ S & R \end{bmatrix} \succ 0$ is equivalent to $R \succ 0$ and $H - S^T R^{-1} S \succ 0$ where $H = H^T$ and $R = R^T$ and S is a matrix with appropriate dimension [9].

3. Main Results

In this section, a state feedback controller will design to make the closed-loop system passive. Control law is $u = Kx$. The following theorem establishes the main result of the state feedback passification.

Theorem 1: If there exist symmetric positive matrices $X \succ 0, Q \succ 0$, matrix Y and scalars $\gamma \geq 0$ and $\varepsilon_i \succ 0,\ i = 1,2,\ldots,6$ such that the following LMI holds

$$\left[\begin{array}{ccc:cccccc} (1,1) & A_1 X & B - XC^T & XE_0^T & Y^T E_2^T & 0 & 0 & XE_3^T & 0 \\ XA_1^T & -Q & -XC_1^T & 0 & 0 & XE_1^T & 0 & 0 & XE_4^T \\ B^T - CX & -C_1 X & -\gamma + \varepsilon_5 H_3 H_3^T + \varepsilon_6 H_4 H_4^T & 0 & 0 & 0 & E_2^T & 0 & 0 \\ \hdashline E_0 x & 0 & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ E_2 Y & 0 & 0 & 0 & -\varepsilon_2 I & 0 & 0 & 0 & 0 \\ 0 & E_1 X & 0 & 0 & 0 & -\varepsilon_3 I & 0 & 0 & 0 \\ 0 & 0 & E_2 & 0 & 0 & 0 & -\varepsilon_4 I & 0 & 0 \\ E_3 X & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I & 0 \\ 0 & E_4 X & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_6 I \end{array}\right] \prec 0 \tag{6}$$

$$\begin{aligned} (1,1) = {} & XA^T + AX + Y^T B^T + BY + Q + \varepsilon_1 H_0 H_0^T \\ & + \varepsilon_2 H_2 H_2^T + \varepsilon_3 H_1 H_1^T + \varepsilon_4 H_2 H_2^T \end{aligned}$$

then delayed system (1) is passive under the control law $u = Kx$ in the sense of definition 2.1 and the state feedback gain can be constructed as
$K = YX^{-1}$

Proof: Choose a Lyapunov–Krasovskii functional as

$$V(x(t)) = x^T(t) P\, x(t) + \int_{t-\tau}^{t} x^T(s) S\, x(s)\, ds \tag{7}$$

We have

$$
\begin{aligned}
&\dot{V}(x(t)) - 2y^T(t)w(t) - \gamma w^T(t)w(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Sx(t) \\
&- x^T(t-\tau)Sx(t-\tau) - 2y^T(t)w(t) - \gamma w^T(t)w(t) \\
&= x^T(t)A^TPx(t) + x^T(t)PAx(t) + x^T(t)Sx(t) + x^T(t)K^TB^TPx(t) + x^T(t)PBKx(t) + x^T(t-\tau)A_1^TPx(t) \\
&+ x^T(t)PA_1x(t-\tau) - x^T(t-\tau)Sx(t-\tau) + w^T(t)B^TPx(t) + x^T(t)PBw(t) - x^T(t)C^Tw(t) - w^T(t)Cx(t) \\
&- x^T(t-\tau)C_1^Tw(t) - w^T(t)C_1x(t-\tau) - \gamma w^T(t)w(t) + x^T(t)(H_0F_0E_0)^TPx(t) + x^T(t)P(H_0F_0E_0)x(t) \\
&+ x^T(t)K^T(H_2F_2E_2)^TPx(t) + x^T(t)P(H_2F_2E_2)Kx(t) + x^T(t-\tau)(H_1F_1E_1)^TPx(t) + x^T(t)P(H_1F_1E_1)x(t-\tau) \\
&+ w^T(t)(H_2F_2E_2)^TPx(t) + x^T(t)P(H_2F_2E_2)w(t) - x^T(t)(H_3F_3E_3)^Tw(t) - w^T(t)(H_3F_3E_3)x(t) \\
&- x^T(t-\tau)(H_4F_4E_4)^Tw(t) - w^T(t)(H_4F_4E_4)x(t-\tau)
\end{aligned}
\tag{8}
$$

It is clear that

$$
\begin{aligned}
&- x^T(t)(H_3F_3E_3)^Tw(t) - w^T(t)(H_3F_3E_3)x(t) - x^T(t-\tau)(H_4F_4E_4)^Tw(t) - w^T(t)(H_4F_4E_4)x(t-\tau) \\
&= x^T(t)[H_3(-F_3)E_3]^Tw(t) + w^T(t)[H_3(-F_3)E_3]x(t) + x^T(t-\tau)[H_4(-F_4)E_4]^Tw(t) - w^T(t)[H_4(-F_4)E_4]x(t-\tau)
\end{aligned}
\tag{9}
$$

and $(-F_i)^T(-F_i) \le I$. Using lemma 2.1, we have

$$
\begin{aligned}
&x^T(t)(H_0F_0E_0)^TPx(t) + x^T(t)P(H_0F_0E_0)x(t) + x^T(t)K^T(H_2F_2E_2)^TPx(t) + x^T(t)P(H_2F_2E_2)Kx(t) \\
&+ x^T(t-\tau)(H_1F_1E_1)^TPx(t) + x^T(t)P(H_1F_1E_1)x(t-\tau) + w^T(t)(H_2F_2E_2)^TPx(t) + x^T(t)P(H_2F_2E_2)w(t) \\
&- x^T(t)(H_3F_3E_3)^Tw(t) - w^T(t)(H_3F_3E_3)x(t) - x^T(t-\tau)(H_4F_4E_4)^Tw(t) - w^T(t)(H_4F_4E_4)x(t-\tau) \le \\
&\varepsilon_1x^T(t)PH_0H_0^TPx(t) + \varepsilon_1^{-1}x^T(t)E_0^TE_0x(t) + \varepsilon_2x^T(t)PH_2H_2^TPx(t) + \varepsilon_2^{-1}x^T(t)K^TE_2^TE_2Kx(t) \\
&+ \varepsilon_3x^T(t)PH_1H_1^TPx(t) + \varepsilon_3^{-1}x^T(t-\tau)E_1^TE_1x(t-\tau) + \varepsilon_4x^T(t)PH_2H_2^TPx(t) + \varepsilon_4^{-1}w^T(t)E_2^TE_2w(t) \\
&+ \varepsilon_5w^T(t)H_3H_3^Tw(t) + \varepsilon_5^{-1}x^T(t)E_3^TE_3x(t) + \varepsilon_6w^T(t)H_4H_4^Tw(t) + \varepsilon_6^{-1}x^T(t-\tau)E_4^TE_4x(t-\tau)
\end{aligned}
\tag{10}
$$

for all $\varepsilon_i \succ 0$, $i = 1,2,\ldots,6$.

Let $\eta = \left[x^T(t), x^T(t-\tau), w(t)\right]^T$, we have

$$
\dot{V}(x(t)) - 2y^T(t)w(t) - \gamma w^T(t)w(t) \le \eta^T M \eta \tag{11}
$$

where $M = M_1 + M_2$ and

$$
M_1 = \begin{bmatrix} (1,1) & PA_1 & PB - C^T \\ A_1^TP & -S & -C_1^T \\ B^TP - C & -C_1 & -\gamma + \varepsilon_5H_3H_3^T + \varepsilon_6H_4H_4^T \end{bmatrix} \tag{12}
$$

$$
\begin{aligned}
(1,1) = {} & A^TP + PA + K^TB^TP + PBK + S \\
& + P(\varepsilon_1H_0H_0^T + \varepsilon_2H_2H_2^T + \varepsilon_3H_1H_1^T + \varepsilon_4H_2H_2^T)P
\end{aligned}
$$

$$
M_2 = \left[\begin{array}{c:cc} \varepsilon_1^{-1}E_0^TE_0 + \varepsilon_2^{-1}K^TE_2^TE_2K + \varepsilon_5^{-1}E_3^TE_3 & 0 & 0 \\ 0 & \varepsilon_3^{-1}E_1^TE_1 + \varepsilon_6^{-1}E_4^TE_4 & 0 \\ 0 & 0 & \varepsilon_4^{-1}E_2^TE_2 \end{array}\right] \tag{13}
$$

To prove the passivity of system (1) we need

$$\dot{V}(x(t)) - 2y^T(t)w(t) - \gamma w^T(t)w(t) \prec 0 \tag{14}$$

because by integrating from (14) we have

$$V(x(t_p)) - V(x(0)) - \gamma \int_0^{t_p} w^T(t)w(t)dt \le 2\int_0^{t_p} y^T(t)w(t)dt \tag{15}$$

then for $x(0)=0$, under initial zero condition, $V(x(0))=0$ and $V(x(t_p)) \ge 0$, hence the system (1) is passive if

$$M = M_1 + M_2 \prec 0 \tag{16}$$

The matrix inequality (16) is not LMI but it is quadratic matrix inequality (QMI). In order to use convex optimization technique to find solutions for this matrix inequality this QMI must be converted to LMI via some transformation. For this purpose define the following transformation matrix

$$T = \begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \tag{17}$$

Multiplying both sides of the matrix in (16) by the matrix T, and denoting $X = P^{-1}$, $Q = P^{-1}SP^{-1}$ and $Y = KX$ yield the linear matrix inequality (18).

$$N = N_1 + N_2 \prec 0 \tag{18}$$

$$N_1 = \begin{bmatrix} (1{,}1)_{N_1} & A_1X & B - XC^T \\ XA_1^T & -Q & -XC_1^T \\ B^T - CX & -C_1X & -\gamma + \varepsilon_5 H_3 H_3^T + \varepsilon_6 H_4 H_4^T \end{bmatrix}$$

$$\begin{aligned}(1{,}1)_{N_1} &= XA^T + AX + Y^T B^T + BY + Q \\ &+ \varepsilon_1 H_0 H_0^T + \varepsilon_2 H_2 H_2^T + \varepsilon_3 H_1 H_1^T + \varepsilon_4 H_2 H_2^T\end{aligned}$$

$$N_2 = \begin{bmatrix} (1{,}1)_{N_2} & 0 & 0 \\ 0 & \varepsilon_3^{-1} X E_1^T E_1 X + \varepsilon_6^{-1} X E_4^T E_4 X & 0 \\ 0 & 0 & \varepsilon_4^{-1} E_2^T E_2 \end{bmatrix}$$

$$(1{,}1)_{N_2} = \varepsilon_1^{-1} X E_0^T E_0 X + \varepsilon_2^{-1} Y^T E_2^T E_2 Y + \varepsilon_5^{-1} X E_3^T E_3 X$$

(18) is equivalent to (19)

$$N_1 - V^T R^{-1} V \prec 0 \tag{19}$$

where

$$V^T = \begin{bmatrix} XE_0^T & Y^T E_2^T & 0 & 0 & XE_3^T & 0 \\ 0 & 0 & XE_1^T & 0 & 0 & XE_4^T \\ 0 & 0 & 0 & E_2^T & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon_2 I & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon_3 I & 0 & 0 & 0 \\ 0 & 0 & 0 & -\varepsilon_4 I & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon_5 I & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_6 I \end{bmatrix}.$$

Using schur complement lemma, (18) and (19) are equivalent to (20)

$$\begin{bmatrix} N_1 & V^T \\ V & R \end{bmatrix} \prec 0 \tag{20}$$

This completes the proof of theorem.

4. Numerical Example

To verify the above theoretical result, consider the following system:

$$A = \begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.3 & 1 \\ -2 & -1 \end{bmatrix},\ B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$C = [1 \quad -2],\ C_1 = [2 \quad 0.5]$$

$$H_0 = \begin{bmatrix} 0.1 & -0.3 \\ 0.4 & 0.2 \end{bmatrix},\ E_0 = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 0.2 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 0 & 0.1 \\ -0.5 & 0.3 \end{bmatrix},\ E_1 = \begin{bmatrix} 0.4 & 0.1 \\ -0.2 & 0.5 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.1 & 0.6 \\ 0.1 & -0.2 \end{bmatrix},\ E_2 = [0.1]$$

$$H_3 = [0.3],\ E_3 = \begin{bmatrix} 0.4 & -0.1 \\ 0.2 & -0.7 \end{bmatrix}$$

$$H_4 = [0.2],\ E_4 = \begin{bmatrix} 0.1 & 0.3 \\ -0.8 & 0.3 \end{bmatrix},\ F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$$

According to theorem 1, these simulation results are obtained from the solution of a feasibility problem

$$X = \begin{bmatrix} 0.7694 & -0.7721 \\ -0.7721 & 3.0072 \end{bmatrix},\ Q = \begin{bmatrix} 5.2867 & -1.4954 \\ -1.4954 & 3.6278 \end{bmatrix},$$

$$Y = [5.4291 \quad -6.6823],\ K = [6.5013 \quad -0.5529]$$

5. Conclusions

The problem of robust passification of linear time-delayed systems with time-varying norm-bounded parameter uncertainties has been studied. LMI results on state feedback controller design are developed. Then controller gains are determined by solving a set of coupled LMIs.

References

[1] R. Lozano, B. Brogliato, O. Egeland, and B. Maschke, Dissipative systems analysis and control: Theory and applications. London, U.K.: Springer-Verlag, 2000.

[2] S.I. Niculescu and R. Lozano, 2001, On the passivity of linear delay systems, IEEE Trans. Auto. Contr. 46 (3), 460-464.

[3] E. Fridman and U. Shaked, 2002, On delay-dependent passivity, IEEE Trans. Auto. Contr. 47 (4), 664-669.

[4] M.S. Mahmoud and A. Ismail, 2004, Passivity and passification of time-delay systems, J. Math. Anal. Appl. 292, 274-258.

[5] C. Li, H. Zhang and X. Liao, 2005, Passivity and passificaion of uncertain fuzzy systems, IEE Proc.-Circuits, Device and Systems 152 (6), 649-653.

[6] D. Peaucelle, A. Fradkov and B. Andrievsky, 2005, Robust passification via static output feedback-LMI results, Proc. 16th IFAC World Congress on Automatic Control. Prague, 2005

[7] D. Peaucelle, A. Fradkov, and B. Andrievsky, 2008, Passification-based adaptive control of linear systems: Robustness issues. Int. J. of Adaptive Control and Signal Processing, 22(6):590–608. doi: 10.1002/acs.1009.

[8] P. Pakshin, D. Peaucelle, 2009, Stabilization and passification of uncertain systems via static output feedback', IEEE-CCA, St Petersburg.

[9] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory. SIAM Studies in Applied Mathematics, Philadelphia, 1994.

[10] Y. Wang, L. Xie and C. de Souza, 1992, Robust control of a class of uncertain nonlinear systems, Sys. Control Let., 19(2):139–49