A high resolution finite volume method for dam break simulation

H.R.Vosoughifar[1], H.R.Jalalpour[2] M.Tabandeh[3], A.Akbarpour[4]

[1,4]Proffesor Of Civil Engineering Department ,Azad University, Tehran south Branch, Tehran, Iran,[2,3]Master Of Science of Civil Engineering Department,Azad University, Tehran south Branch, Tehran, Iran

Paper Reference Number: 15
Name of the Presenter: H.R.Jalalpour

Abstract

A high resolution finite volume method for solving the shallow water equations with rectangular mesh is developed applying MATLAB software in this paper. The scheme is formally uniformly second order accurate and satisfies maximum principles. The model is verified by comparing the model output with condition of anti-symmetric and circular dam break with documented results. For more investigation we utilized SPSS statistical software. Very good agreement has been achieved in the verification phase. It can be considered as an efficient implement for the computation of shallow water problems, especially concerning those having discontinuities. A simple example of the collapse of water supply reservoir in a valley is used to demonstrate the capability of the model. The presented model is able to resolving shocks, handling, complex geometry, including the influence of steep bed slopes.

Keywords: Dam break, Finite volume method, Local-Lax–Friedrich second order scheme, Rectangular mesh.

1. Introduction

Although safety criteria have been considered in design, construction and operation of dams, dams maybe broken under unpredictable events. Therefore, it is necessary to analyze dam break incident. One of the most important applications of dam break analysis is preparing Emergency Action Plan (EAP). An EAP is a formal plan that determines potential emergency conditions at a dam and prescribes the procedures to be followed to minimize property damage and loss of life. The water surface elevations and travel times resulting from dam break that clearly indicate the potential hazard to downstream lives and property should be presented on an inundation map. The information on the inundation map must be up-to-date and adequate for the development of a workable EAP. In a dam break a large amount of water is accidentally released downstream. For this reason, the numerical model must be able to handle correctly the flow discontinuities, high gradients near drying/wetting front, complex bed geometry, and source terms. A finite-volume method (FVM) is based on the integral form of the equations. The discretization of the integral form of governing equations ensures that the basic quantities (mass and momentum) will also be conserved across a discontinuity. The Local Lax–Friedrich solver enabling one able to handle discontinuous solutions is used to evaluate fluxes at the cell faces. Each of the numerical methods are consist of two steps, discretization and discreted equations solution, that selected method influence the precision of the final results. There are some discretization methods such as finite element, finite difference and finite volume that finite volume method is commonly used because of its compatibility with physical geometries and modeling the complicated topographies exactly. In FVM the governing equations are integrated over the control volume. This allows handling of discontinuities in the solution. Patankar (1972) used finite volume method for numerical solving of heat transfer equations. Hirsch (1988) used the discretization of the integral form of governing equations. This issue ensures that the basic quantities (mass and momentum) will also be conserved across a discontinuity. Shu and Osher (1988) brought together the time discretization by the third-order total variation diminishing (TVD) Runge–Kutta method .Chaudhry (1993) utilized shock capturing methods to model one dimensional free surface flows . Satya et al. (1998) proofed that the FVM is quite flexible and can be readily used on arbitrary meshes .Valiani et al. (2002) demonstrated that by conserving the basic quantities (mass and momentum), the shock-capturing property will be satisfied. Yoon and Kang (2004) used the HLL approximate Riemann solver in order to evaluate fluxes at the cell faces. The friction terms are treated in a fully implicit manner by an operator splitting technique to prevent numerical instabilities caused by small water depth near the dry zones.

2. Research Methodology

2-1.Governing equation

The 2D shallow water equations with source terms may be written in vector form,

$$U_t + F_x + G_y = S \tag{1}$$

$$U_t = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix} \tag{2}$$

$$F_x = \begin{bmatrix} q_x \\ \frac{{q_x}^2}{h} + \frac{1}{2} g h^2 \\ \frac{q_x q_y}{h} \end{bmatrix} \tag{3}$$

$$G_y = \begin{bmatrix} q_y \\ \frac{q_x q_y}{h} \\ \frac{{q_y}^2}{h} + \frac{1}{2} g h^2 \end{bmatrix} \tag{4}$$

$$S = \begin{bmatrix} 0 \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{bmatrix} \tag{5}$$

where *U* is the vector of conserved variables, *F* and *G* are the flux vector functions and *S* is the vector of source terms, where *u* and *v* are velocity components in the *x* and *y* directions, respectively; *h* is water depth is acceleration due to gravity; (s_{0x},s_{0y}), are bed slopes in the *x* and *y* directions, and (s_{fx},s_{fy}) are friction slopes in the *x* and *y* directions, respectively. In this study, the friction slopes are estimated by using the Manning formulas:

$$S_{fx} = \frac{n^2 u\sqrt{u^2+v^2}}{h^{1.33}} \quad , \quad S_{fy} = \frac{n^2 v\sqrt{u^2+v^2}}{h^{1.33}} \tag{6}$$

where *n* = Manning's roughness coefficient. In the case of dam-break flow, the influence of bottom roughness prevails over the turbulent shear stress between cells. Therefore the effective stress terms were neglected in the computation.

2-2.Governing equation discretization

In this section, Discretization of the equation 1 is performed applying finite volume method with unstructured rectangular mesh in domain ***Ω*** (figure 1).It should be mentioned that F face vertexes nomination direction is counterclockwise from b to a centering o in fig1.

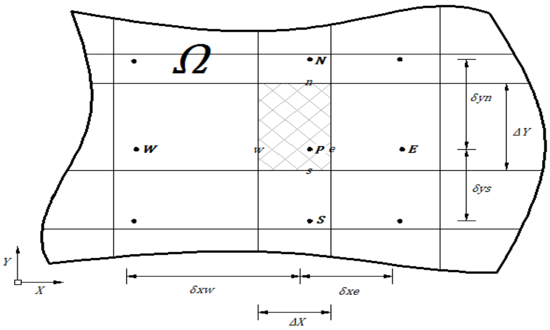

Figure1: studied domain ***Ω***

$$\frac{\partial}{\partial t}\int_v u\, d_v + \int_v \vec{\nabla}. H(u) d\vartheta = \int_v S(u) d_v \tag{7}$$

Where $H(u)$ is input and output flux to rectangular cell that contains G (u) and F (u) functions in x and y directions.
By implementing divergence theorem:

$$\int_{\mathrm{v}} \vec{\nabla}. \mathrm{H(u)} d\vartheta = \oint_{\Omega} n. \mathrm{H(u)} d\Omega \tag{8}$$

$$\frac{\partial}{\partial t}\int_{\mathrm{v}} \mathrm{u\ d_v} + \int_{\Omega} \mathrm{n. H(u)\ d_\Omega} = \int_{\mathrm{v}} \mathrm{S(u) d_v} \tag{9}$$

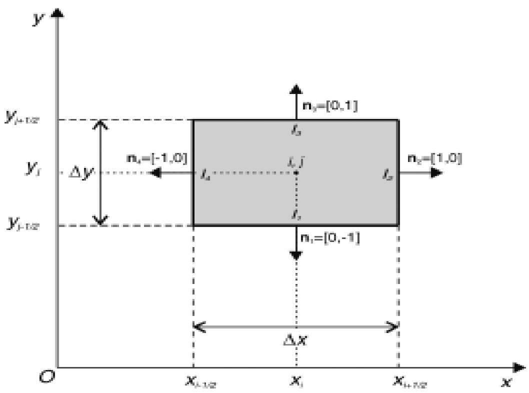

Figure2: A rectangular cell in x, y direction

$$\mathbf{U}_{i,j}^{n} \approx \frac{1}{|I_{i,j}|}\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathbf{U}\left(x,y,t^{n}\right)dx\,dy \tag{10}$$

$$\frac{d}{dt}\mathbf{U}_{i,j} = -\frac{1}{|I_{i,j}|}\sum_{s=1}^{4}\mathcal{F}_s \tag{11}$$

$$\Delta x = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}} \tag{12}$$

$$\Delta y = y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}} \tag{13}$$

$$|I_{i,j}| = \Delta x \times \Delta y \tag{14}$$

$$\mathcal{F}_s = \int_{l_s}\left[n_1\mathbf{F}\left(\mathbf{U}\right) + n_2\mathbf{G}\left(\mathbf{U}\right)\right]dl \tag{15}$$

$$\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^{n} - \frac{1}{|I_{i,j}|}\sum_{s=1}^{4}\int_{t^n}^{t^{n+1}}\mathcal{F}_s\,dt \tag{16}$$

$$\begin{aligned}
\int_{t^n}^{t^{n+1}}\mathcal{F}_1\,dt &= \int_{t^n}^{t^{n+1}}\int_{x_{i+\frac{1}{2}}}^{x_{i-\frac{1}{2}}} -\mathbf{G}\Big(\mathbf{U}\big(x,y_{j-\frac{1}{2}},t\big)\Big)dx\,dt \approx -\mathbf{G}_{i,j-\frac{1}{2}}\Delta x\Delta t,\\
\int_{t^n}^{t^{n+1}}\mathcal{F}_2\,dt &= \int_{t^n}^{t^{n+1}}\int_{y_{i+\frac{1}{2}}}^{y_{i-\frac{1}{2}}} \mathbf{F}\Big(\mathbf{U}\big(x_{i+\frac{1}{2}},y_j,t\big)\Big)dy\,dt \approx \mathbf{F}_{i+\frac{1}{2},j}\Delta y\Delta t,\\
\int_{t^n}^{t^{n+1}}\mathcal{F}_3\,dt &= \int_{t^n}^{t^{n+1}}\int_{x_{i+\frac{1}{2}}}^{x_{i-\frac{1}{2}}} \mathbf{G}\Big(\mathbf{U}\big(x,y_{j+\frac{1}{2}},t\big)\Big)\,dx\,dt \approx \mathbf{G}_{i,j+\frac{1}{2}}\Delta x\Delta t,\\
\int_{t^n}^{t^{n+1}}\mathcal{F}_4\,dt &= \int_{t^n}^{t^{n+1}}\int_{y_{i+\frac{1}{2}}}^{y_{i-\frac{1}{2}}} -\mathbf{F}\Big(\mathbf{U}\big(x_{i-\frac{1}{2}},y,t\big)\Big)\,dy\,dt \approx -\mathbf{F}_{i-\frac{1}{2},j}\Delta y\Delta t,
\end{aligned} \tag{17}$$

Final discreted equation can be written as:

$$U^{n+1} = U^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2} - F_{i-1/2}\right]^n - \frac{\Delta t}{\Delta y}\left[G_{j+1/2} - G_{j-1/2}\right]^n + S^n\Delta t \tag{18}$$

$$f^*(u_i^n,u_{i+1}^n) = F_{i+\frac{1}{2},j} \text{ ، } f^*(u_{i-1}^n,u_i^n) = F_{i-\frac{1}{2},j} \tag{19}$$

$$g^*\big(v_{j-1}^n,v_j^n\big) = G_{i,j-\frac{1}{2}} \text{ ، } g^*\big(v_j^n,v_{j+1}^n\big) = G_{i,j+\frac{1}{2}} \tag{20}$$

2-3.The Local Lax-Friedrichs high order scheme in rectangular mesh

$$f^*(u_i^n,u_{i+1}^n) = \frac{f(u_i^n)+f(u_{i+1}^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{u_i^n+u_{i+1}^n}{2}\right)\right|(u_{i+1}^n - u_i^n) \tag{21}$$

$$f^*(u_{i-1}^n,u_i^n) = \frac{f(u_i^n)+f(u_{i-1}^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{u_i^n+u_{i-1}^n}{2}\right)\right|(u_i^n - u_{i-1}^n) \tag{22}$$

$$\lambda = \bar{u} + \sqrt{g\bar{h}} \tag{23}$$

$$g^*\big(v_j^n,v_{j+1}^n\big) = \frac{g\big(v_j^n\big)+g(v_{j+1}^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{v_j^n+v_{j+1}^n}{2}\right)\right|(v_{j+1}^n - v_j^n) \tag{24}$$

$$g^*\big(v_{j-1}^n,v_j^n\big) = \frac{g\big(v_j^n\big)+g(v_{j-1}^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{v_j^n+v_{j-1}^n}{2}\right)\right|(v_j^n - v_{j-1}^n) \tag{25}$$

$$\lambda = \bar{v} + \sqrt{g\bar{h}} \tag{26}$$

After computing intercell flux by utilizing Local Lax-Friedrichs scheme, the equation can be solved and the final result can be calculated after each time step. The Δt value should compute by using courant Friedrichs lewy (CFL) after each time step as follow:

$$\Delta t = CFL * min\left(\frac{\Delta\varepsilon}{max|\lambda_1|}, \frac{\Delta\varepsilon}{max|\lambda_2|}\right) \tag{27}$$

$$\lambda_1 = \bar{u} + \sqrt{g\bar{h}} \tag{28}$$

$$\lambda_2 = \bar{v} + \sqrt{g\bar{h}} \tag{29}$$

3. Results and Analysis

3-1.Validation

The numerical model is validated using the example of dam break for which an analytical solution is available.

3-1-1.Two dimensional anti-symmetric dam break test

Many scientists are implemented this test such as Fennena, Chaudhry and Navarro. The channel that is located in horizontal bed has 200 meters length, 200 meters width and it contains an anti-symmetric cut with 75 meters width. The channel and dam domain are assumed frictionless and the upstream water depth is proposed 10 meters and the downstream water depth is assumed 5 meters. In this test we proposed 40*40 node points and the total time for comparing values is suggested 7.2 second.

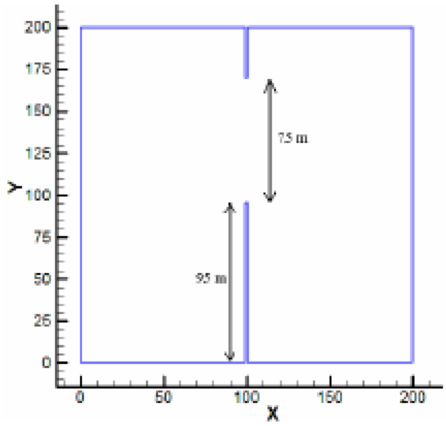

Figure3: Dam break model

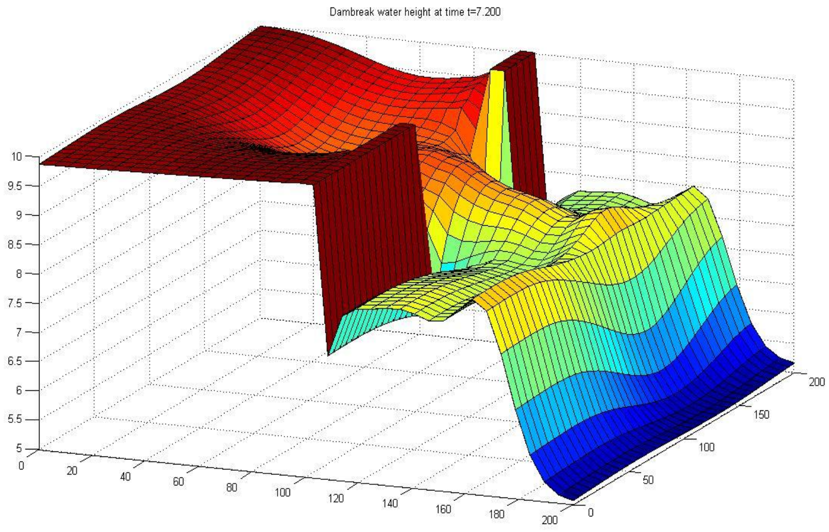

Figure4: Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second based on presented model

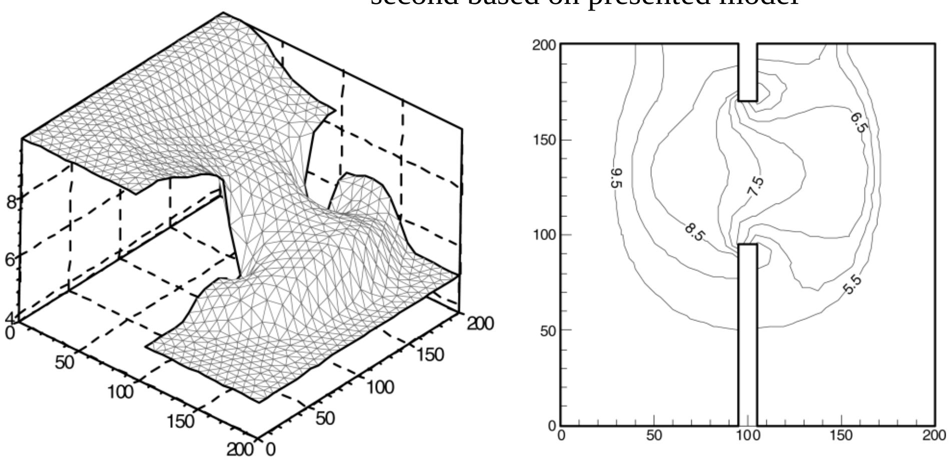

Figure5: Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second

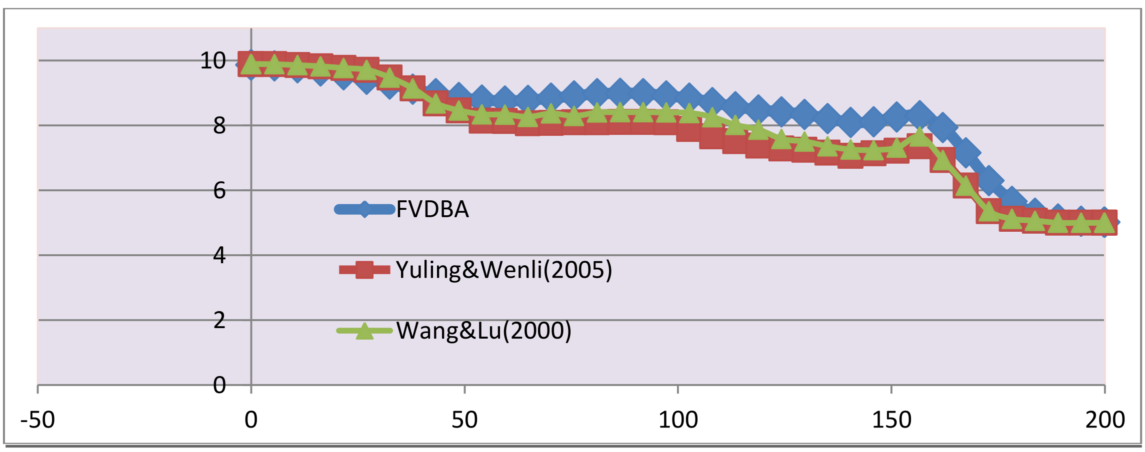

Figure6: Total results of comparing MATLAB code output with references in two different meshes

Wang&Lu	Yuling&Wenli	Reference
0.97	0.95	P-VALUE

Table1.P-VALUE resulted from statistical t test

3-1-2.Two dimensional dam break circular test

In this test we considered a frictionless and horizontal rectangular domain that has 200 meters length and 200 meters width. The initial conditions consist of two states separated by a circular discontinuity. The computational grid consists of 40 *40 cells and the radius of the circle r = 50 meters and it is centered at x = 100 meters. The water depth outside the circle is 1 meter deep and inside the circle is 10 meters deep. The water depth has shown after 2 second. This result demonstrates that the model is able to simulate complicated geometries.

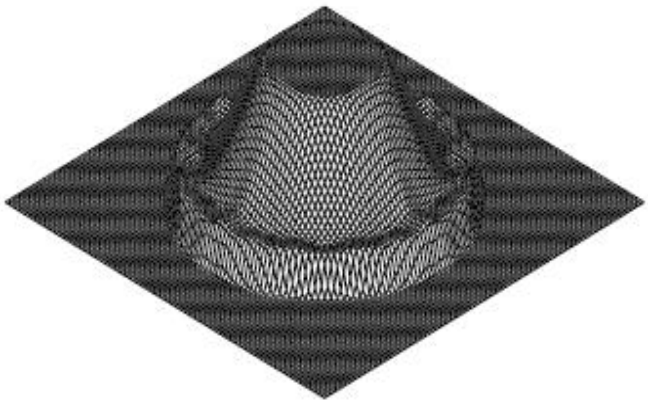

Figure7:Two-dimensional solution of the circular dam break problem in a rectangular domain after 2 second based on references

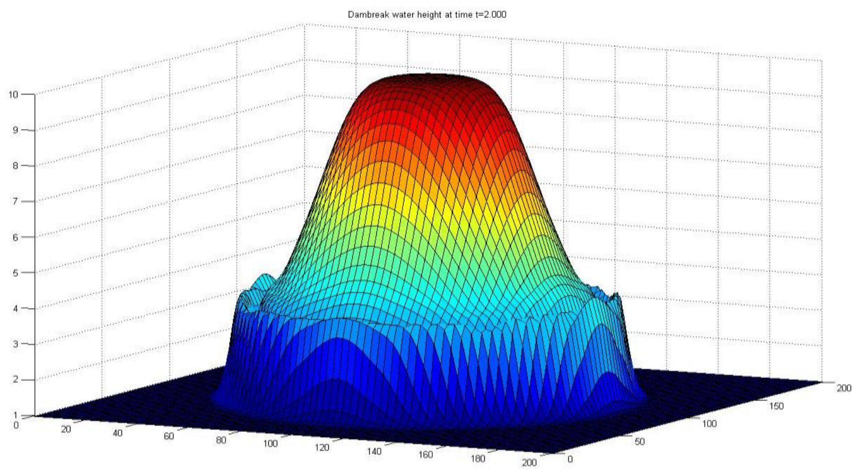

Figure 8:Two-dimensional solution of the circular dam break problem in a rectangular domain after 2 second based on presented model

Zoppou	Reference
0.93	P-VALUE

Table2.P-VALUE resulted from statistical t test

4. Conclusions

-In presented research it is shown that Lax–Friedrich scheme with finite volume and rectangular mesh is a suitable combination in order to simulate two dimensional dam break problem. The advantages of this method are very hopeful especially in reconstructing the introduced tests outputs and precise solving.

-In any test, significant numerical dispersion problem or nonphysical alternation does not observe in results .This consequence was predicted because by combination of finite volume and Lax–Friedrich scheme other results were not expectable.

-The computed P-VALUES are near to 1 that demonstrates high ability of presented mathematical-numerical model in dam break evaluation.

-Finite volume method have so many advantages rather than other numerical methods, finite element, such as suitable compatibility with studied domain specially in dam break problem, easy prescription for seepage equation discretization.

-Verified mathematical-numerical model is capable to model different environmental conditions such as heterogeneous and anisotropic porous media with complicated topographies. This model not only have specific flexibility for meshing such as changing number of elements at sensitive and important points in investigated domain but also have easy understanding and users application.

-This model not only shows every point's depth and interval but also gives velocity in each time step.

- Riemann boundary condition is used in order to model the boundary conditions exactly. The reason is that precise head value in some studied domain boundaries is not specified.

References

Aliparast.M.(2009).*Two-dimensional finite volume method for dam-break flow simulation*. International Journal of Sediment Research 24 (2009) 99–107.

Chaudhry, M. H. (1993).*Open-Channel Flow*.Prentice-Hall, Englewood Cliffs, New Jersey.

Fennena ,R.J.,Chaudhry,M.H. (1986). *Explicit Numerical Schemes for Unsteady Free-Surface Flows with Shocks*. Water Resources Research, Vol.22, n.13.

Garcia-Navarro, P.,Brufau P. (1992). *One-Dimensional Dam Break Flow Modeling: Some Results.*

Hirsch C. (1988). *Numerical Computation of Internal and External Flows. Vol. 1: Fundamentals of Numerical Discretization,Wiley, Chichester, England.*

Leveque .Randall J. (2004). *Finite Volume Methods For Hyperbolic Problems*.

Loukili.Youssef, Soula¨ımani.Azzeddine. (2007). *Numerical Tracking of Shallow Water Waves by the Unstructured Finite Volume WAF Approximation*. International Journal for Computational Methods in Engineering Science and Mechanics, 8:1–14.

Murthy Jayathi Y. (2002) . *Numerical Methods in Heat, Mass, and Momentum Transfer*.

Patankar. S. V. (1980). *Numerical heat transfer and fluid flow*. McGraw-Hill, New York.

Satya Sai B. V. K., Zienkiewicz O. C., Manzari M. T., Lyra P. R. M., Morgan K. (1998). *General purpose versus special algorithms for high-speed flows with shocks.* International Journal for Numerical Methods Fluids, 27, pp. 57–80.

Shu C. W, Osher S. (1988). *Efficient implementation of essentially non-oscillatory shock capturing schemes*.Journal of Computational Physics, 77, pp. 439–471.

Valiani A., Caleffi V., Zanni A. (2002). *Case study: Malpasset dam-break simulation using a two-dimensional finite volume method*. Journal of Hydraulic Engineering, Vol. 128, No. 5, pp. 460–472.

Yoon T. H. and Kang S. (2004). *Finite volume model for two-dimensional shallow water flows on unstructured grids.Journal of Hydraulic Engineering*. Vol. 130, No. 7, pp. 678–688.

Zoppou .Christopher, Roberts .Stephen. (2000). *Numerical solution of the two dimensional unsteady dam break.*