


# Assessment the effect of various meshing in finite volume precision in order to numerical solution of seepage diffusion equation beneath the Roller Compacted Concrete dam

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**Abstract**

Presenting a suitable numerical solution for modeling seepage phenomenon as one of the destruction factors that have significant affect on dam's stability, is necessary. Between several kinds of numerical methods, finite volume method has so many advantages for simulating different fluid's flow problems with complicated conditions. Different kinds of meshes such as structured and unstructured mesh can be used in finite volume method that implementing suitable mesh has considerable affect on the results of finite volume method by modeling the complicated topographies actually and increasing the computations accuracy. Therefore appropriate choice of meshing is one of the most important factors in numerical solution. In this article, first finite volume method with different kinds of meshes such as rectangular, orthogonal triangular, non orthogonal triangular and voronoi is utilized for discretization seepage differential equation.Second,powerful programming software, MATLAB, is implemented for writing flexible program which is able to execute mathematical-numerical model. In order to evaluate the effects of various meshes in final result's accuracy, seepage problem is modeled beneath several RCC dams by applying presented MATLAB code. Riemann boundary condition is used in this code in order to model the boundary conditions precisely. These cases are also simulated in PHASE2 7.0(2010) software for validation the model results. Then the results of the model and reference are compared by executing statistical software, SPSS17 and their compatibility proportion is evaluated. The results of comparison indicates that type of the meshing is affected P-VALUE and is varied this index more than 0.1.The results of model with finite volume and reference with finite element method had no statistical difference (P-VALUE>0.05).

**Keywords:** seepage, finite volume method, various meshing, increasing accuracy, Riemann boundary condition.

## 1. Introduction

The problem of seepage flow underneath of gravity dams can be formulated in terms of a non-linear partial differential equation. Although empirical formulations are proposed for simple cases, due to inherently complex boundary conditions and intricate physical geometries in any practical problem, an analytical solution is not possible for complicated dam foundations. Thus today numerical solutions are commonly implemented. There are some discretization methods such as finite element, finite difference and finite volume that finite volume method is commonly used because of its compatibility with physical geometry of porous media and modeling the complicated topographies exactly. Mesh is required in order to discrete the verified domain which is categorized into two approaches, namely, structured or unstructured mesh. Type of the chosen mesh has wide effect on results precision and computations rate. Triangular unstructured mesh has some advantages such as compatibility with complex geometries. Voronoi unstructured mesh is a modern method that has so many advantages such as flexibility for calculating the fluid flow over meshes precisely, modeling the cases very close to prototype and increasing the accuracy of the results. Darcy (1856) performed some experimental researches on fluid flow over soil and filters and presented a simple formulation between fluid velocity and hydraulic gradient. Patankar (1972) firstly used Finite volume method for numerical solving of heat transfer equations. He (1978-1980) investigated fluids flow problems with finite volume method. Uromeihy and Barzegari (2004) brought together plaxis software and triangular mesh for computing seepage in chaparabad dam. HongWei Xie (2005) studied impermeable layer in RCC dams and verified the type and thickness of this layer. Shamsai and Vosoughifar (2005) brought together finite volume discretization method with rectangular and triangular mesh for investigating nitrate movements in groundwater. Ashley Pitcher (2005) assumed steady state fluid flow and homogeneous and isotropic porous media .Then investigated seepage in dam applying two dimensional finite element methods with triangular three nodes and triangular six nodes meshes. Sabbagh yazdi (2007) evaluated three dimensional galerkin finite volume explicit solvers for solving seepage and uplift in concrete gravity dam foundation in heterogeneous and isotropic porous media including three incline layers. They utilized tetrahedrons elements with four nodes. Bazrafshan (2008) investigated seepage problem considering nonlinear permeability coefficient implementing finite difference method. Zu-jiang Luo (2008) presented a numerical model in order to control seepage in three dimensional state beneath the deep dam foundations applying finite element method.He Yo (2009) evaluated different affecting factors on concrete gravity dam's seepage problem such as cut-off wall, the permeability coefficient of cut-off wall, drainage holes and grout curtains. They solved seepage diffusion equation applying finite element method and compared the effectiveness of the seepage control system before and after using them. Jun-feng fu and Sheng jin (2009) brought together a numerical model to simulate seepage flow in unsteady state with both saturation and water head as variables to describe the seepage domain.Shen Zhenzhong (2009) studied seepage applying finite element method that contains two methods, mid-section method and equivalent nodal method. PEI Guihong (2010) presented a numerical model to investigate the effect of water head change on slope stability.

## 2. Research Methodology

### 2-1.Modeling algorithm

The studied domain $\boldsymbol{\Omega}$ is discretized applying four kinds of meshes such as rectangular, orthogonal triangular, non orthogonal triangular and voronoi that is presented at the first time by Peter Lijeune Dirichlet in 1850.Voronoi corrected these meshes after half century. Triangular mesh is presented by delaunay in 1934 and it is based on Dirichlet method .Delaunay triangulation created by several algorithms such as Watson and Bowyer Algorithm, Step By Step Algorithm .In this article triangulation is created applying Qhull program in MATLAB software. Voronoi mesh have several characteristics such as: The chosen point has lower distance in devoted domain rather than other points. If one point has the same distance from several domains, it will be divided between domains. Indeed these points create voronoi cells boundaries. Consequently internal sections of voronoi mesh are consisting of nodes which belong to one domain and boundaries include nodes that belong to several domains.

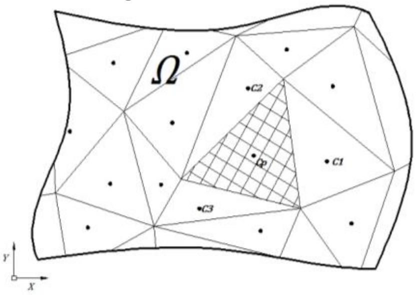


**Figure1-1**: Triangular mesh

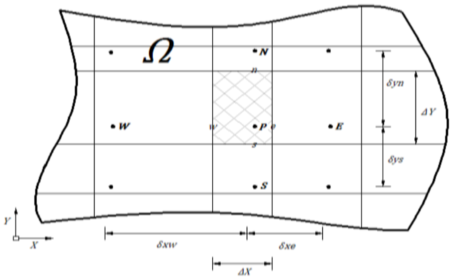


**Figure1-2**: Rectangular mesh

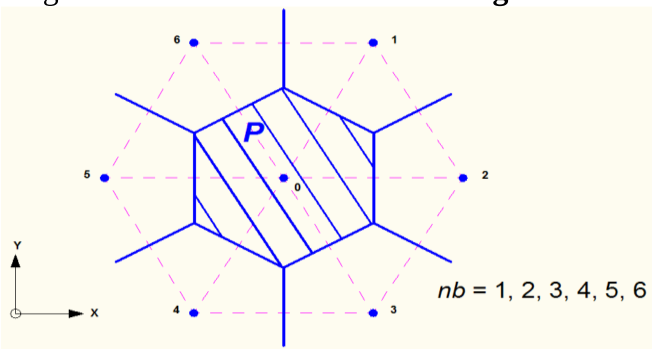


**Figure1-3**: Voronoi mesh

**Figure1**: Studied domain $\boldsymbol{\Omega}$ in different meshes

In order to solve discreted equations, $\boldsymbol{\varphi}$ value in each node compute considering its discreted equation and newest adjacent nodes $\boldsymbol{\varphi}$ values. Solution procedure can be expressed as:

I. Initial guess for $\boldsymbol{\varphi}$ value in every node as initial conditions.
II. Calculating $\boldsymbol{\varphi}$ value in investigated node considering its discreted equation.
III. Performing pervious step for all nodes in studied domain ($\boldsymbol{\Omega}$), one cycle is performed by repetition this step.
IV. Verifying convergence clause. If this clause satisfied the computing will end otherwise the computations will be repeated from step II.

**2-2.Governing mathematical model**

General differential equation can be described as equation 1 considering conservation law for dependent variable $\boldsymbol{\varphi}$ that introduces quantities such as mass, momentum, heat, ect.

$$\frac{\partial}{\partial t}(\rho\varphi) + div(\rho\vec{u}\varphi) = div\left(\vec{\Gamma}\ grad\ \varphi\right) + S \tag{1}$$

Equation 1 components from left to right are unsteady term, convection term, diffusion and source term. Where, $\vec{\Gamma}$ is diffusion coefficient,S is the component of source term, $\rho$ is density and $\vec{u}$ is the velocity vector. The seepage differential equation in steady state without source or sink can be written as:

$$div(\vec{k}\ grad\ h) = 0 \tag{2}$$

Where K is soil permeability coefficient and h is head in investigated nodes.

## 2-3.Governing equation discretization

In this section, Discretization of the equation 2 is performed applying finite volume method with rectangular, orthogonal triangular, non orthogonal triangular and voronoi mesh in domain ***Ω*** (figure 2) and discrete equation with voronoi mesh can be modeled as:

$$a_p h_p = \sum_f (a_{nb} h_{nb})_f \tag{3}$$

The components of equation 3 are:

$$a_{nb} = Df = \left(\frac{k.A}{\Delta\varepsilon}\right)_f \tag{4}$$

$$a_P = \sum_{nb} a_{nb} \tag{5}$$

Where the above formulations parameters define as follows:

$$\Delta_\varepsilon = \sqrt{(x_{nb} - x_p)^2 + (y_{nb} - y_p)^2} \tag{6}$$

$$\varepsilon_x = \frac{(x_{nb} - x_p)}{\Delta\varepsilon} \tag{7}$$

$$\varepsilon_y = \frac{(y_{nb} - y_p)}{\Delta\varepsilon} \tag{8}$$

$$k_f = (k_x \varepsilon_x + k_y \varepsilon_y)_f \tag{9}$$

Seepage discrete equation with triangular mesh can be written as:

$$a_p h_p = \sum_{nb} (a_{nb} h_{nb}) + b \tag{10}$$

$$a_{nb} = \left(\frac{k_f}{\Delta_\varepsilon}\ \frac{A_f.A_f}{A_f.e_\varepsilon}\right)_{nb}, \quad nb = 1, 2, 3 \tag{11}$$

$$a_P = \sum_{nb} a_{nb} \tag{12}$$

$$b = -\sum_{nb} (\rho_f)_{nb} \quad , \quad nb = 1, 2, 3 \tag{13}$$

$$\rho = \left(\frac{(\vec{k}.*\vec{A}).\vec{A}}{\vec{A}.\vec{e_\varepsilon}} \vec{e_\eta}.\vec{e_\varepsilon}.\varphi_\eta\right)_f \tag{14}$$

$$\vec{e_\varepsilon} = \left[\frac{(x_{C1} - x_{Cp})}{\Delta\varepsilon}, \frac{(y_{C1} - y_{Cp})}{\Delta\varepsilon}\right] \tag{15}$$

$$\vec{e_\eta} = \left[\frac{(x_b - x_a)}{\Delta\eta}, \frac{(y_b - y_a)}{\Delta\eta}\right] \tag{16}$$

$$\varphi_\eta = \frac{\varphi_b - \varphi_a}{\Delta\eta} \tag{17}$$

In rectangular mesh seepage discrete equation can be modeled as:

$$a_p h_p = \sum_f (a_{nb} h_{nb})_f \tag{18}$$

$$a_P = \sum_{nb} a_{nb} \quad , \quad a_{nb} = \begin{cases} \Gamma_{x_e} \frac{\Delta y}{\delta_{x_e}} & \text{for east neighbor} \\ \Gamma_{x_w} \frac{\Delta y}{\delta_{x_w}} & \text{for west neighbor} \\ \Gamma_{y_n} \frac{\Delta x}{\delta_{y_n}} & \text{for north neighbor} \\ \Gamma_{y_s} \frac{\Delta x}{\delta_{y_s}} & \text{for south neighbor} \end{cases} \tag{19}$$

**2-4. numerical-mathematical modeling**

In this step, MATLAB software is utilized in order to write finite volume program which is able to model seepage problem. This program cans discrete domain ***Ω*** applying voronoi mesh with different geometry and boundary conditions and arbitrary vertices. Then it can solve algebraic equations system which is resulted from discretization. At the end total hydraulic head calculate in nodes and stream lines and potential lines draw. Finally, PHASE2 7.0(2010) software which its accurate results is demonstrated in several papers, is implemented as reference in order to evaluate numerical-mathematical model results precision. PHASE2 7.0 discretization method is Gauss Elimination iterative method that is based on finite element method. At the end, the model and reference results are compared by statistical software, SPSS17.

**3. Results and Analysis**

**3-1.Case study**

In this step, seepage flow is investigated beneath the roller compacted concrete dam with different kinds of meshes such as rectangular, orthogonal triangular, non orthogonal triangular and voronoi mesh. It should be mentioned that every prepared numerical-mathematical model and reference characteristics such as studied domain geometry, environmental conditions and head values in upstream and downstream of the dam, are equal. The initial head values of control volumes acquire with supposition equipotential lines. The numbers of control volumes are 120. The case study is a roller compacted concrete dam that its characteristics are shown in figure 2.The dam body assumed impermeable and the environmental conditions were homogeneous and anisotropic with wet downstream.

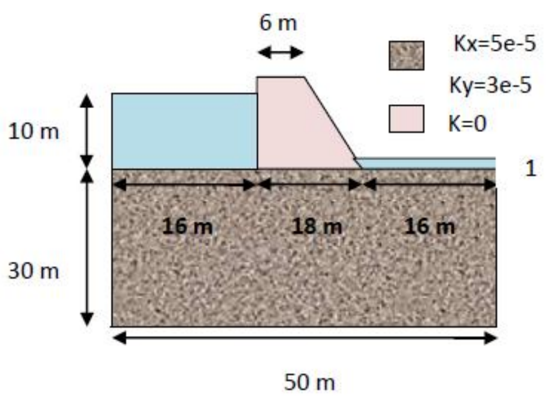


**Figure2**: Geometry condition of studied dam

**3-2.Numerical results**

Several results of four kinds of meshes are presented below. It should be mentioned that PHASE2 program and MATLAB prepared code total heads are gather. These values are chosen randomly.

| MATLAB total head | Phase2 total head | Y | X | MATLAB total head | Phase2 total head | Y | X |
|---|---|---|---|---|---|---|---|
| 33.72 | 32.97 | 17.98 | 45.75 | 35.53 | 37.10 | 0.00 | 0.00 |
| 25.13 | 25.51 | 23.94 | 25.00 | 34.43 | 34.15 | 0.10 | 41.60 |
| 32.32 | 31.03 | 29.90 | 37.45 | 35.21 | 35.97 | 6.06 | 20.85 |
| 35.13 | 35.55 | 30.00 | 25.00 | 35.06 | 35.53 | 12.02 | 25.00 |

Table1-Random values of rectangular mesh for evaluating results precision

| MATLAB total head | Phase2 total head | Y | X | MATLAB total head | Phase2 total head | Y | X |
|---|---|---|---|---|---|---|---|
| 33.40 | 32.97 | 17.98 | 45.75 | 35.51 | 37.10 | 0.00 | 0.00 |
| 33.95 | 35.51 | 23.94 | 25.00 | 35.21 | 34.15 | 0.10 | 41.60 |
| 31.55 | 31.03 | 29.90 | 37.45 | 35.80 | 35.98 | 6.06 | 20.85 |
| 32.61 | 35.55 | 30.00 | 25.00 | 35.32 | 35.53 | 12.02 | 25.00 |

Table2-Random values of orthogonal triangular mesh for evaluating results precision

| MATLAB total head | Phase2 total head | Y | X | MATLAB total head | Phase2 total head | Y | X |
|---|---|---|---|---|---|---|---|
| 33.98 | 32.83 | 18.98 | 46.98 | 35.52 | 37.10 | 0.00 | 0.00 |
| 33.75 | 35.42 | 24.94 | 25.33 | 34.99 | 35.95 | 7.06 | 21.18 |
| 31.72 | 31.03 | 29.90 | 37.45 | 34.82 | 34.71 | 5.06 | 32.96 |
| 32.60 | 35.55 | 30.00 | 25.00 | 34.51 | 35.48 | 13.02 | 25.33 |

Table3-Random values of non orthogonal triangular mesh for evaluating results precision

| MATLAB total head | Phase2 total head | Y | X | MATLAB total head | Phase2 total head | Y | X |
|---|---|---|---|---|---|---|---|
| 35.48 | 35.54 | 17.04 | 24.90 | 31.09 | 31.03 | 0.00 | 0.00 |
| 32.97 | 32.52 | 21.32 | 45.65 | 31.03 | 31.02 | 8.67 | 29.25 |
| 35.14 | 35.03 | 29.90 | 41.60 | 40.00 | 40.00 | 8.47 | 33.20 |
| 35.45 | 35.52 | 30.00 | 25.00 | 35.48 | 35.55 | 12.95 | 25.10 |

Table4-Random values of voronoi triangular mesh for evaluating results precision

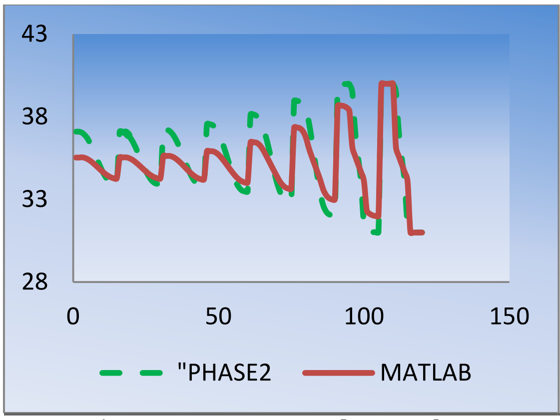


**Figure3-1**: rectangular mesh

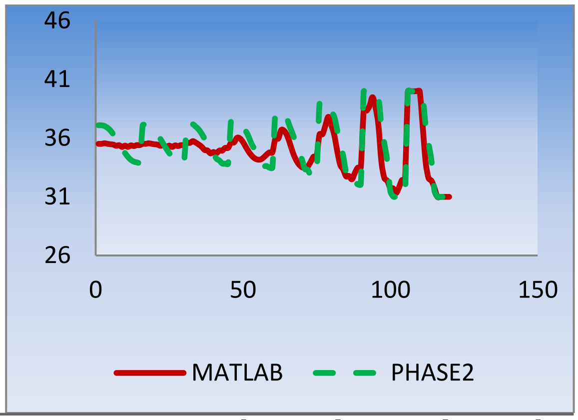


**Figure3-2**: non orthogonal triangular mesh

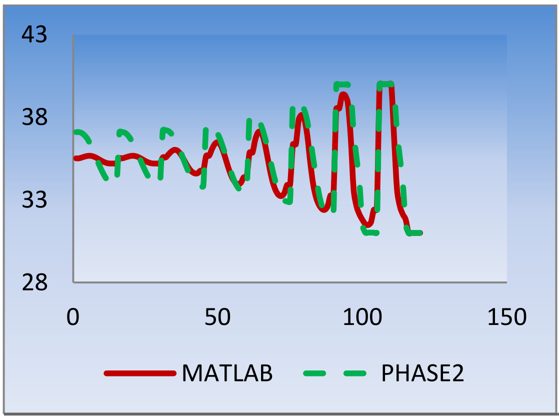


**Figure3-3**: orthogonal triangular mesh

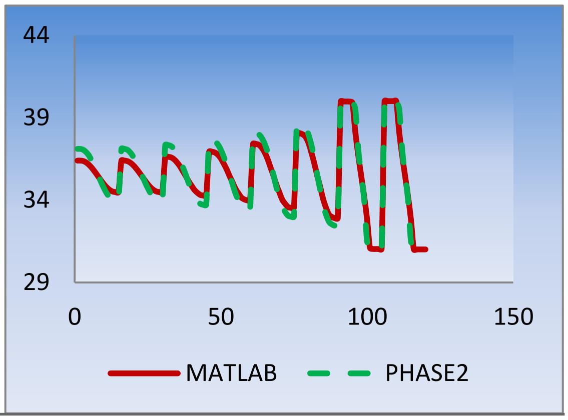


**Figure3-4**: voronoi mesh

**Figure3**: Comparing total head values in different meshes

P-VALUE index is computed applying statistical test SPSS17 and comparing total head in 120 control volumes in order to assess more accurate results. The P-VALUE indexes are shown in table5.If computed P-VALUE index be closer to 1, the compared values have better compatibility while the less than 0.05 P-VALUES indicated that introduced MATLAB and PHASE2 results had significant statistical difference.

| Voronoi | Non orthogonal triangular | Orthogonal triangular | Rectangular | Kind of mesh |
|---|---|---|---|---|
| 0.986 | 0.879 | 0.897 | 0.938 | P-VALUE |

Table5. P-VALUE indexes

## 4. Conclusions

-Comparing computed P-VALUES of different kinds of meshes indicates that voronoi mesh have most compatibility and precision with reference results. Rectangular mesh, orthogonal triangular mesh and non orthogonal triangular mesh have lower precision.

-Total computed heads in MATLAB and PHASE2 are very close together applying voronoi mesh. The P-VALUE with voronoi mesh is equal 0.986 that is very close to 1.This issue demonstrate that voronoi mesh have high precision in calculating total head values beneath the RCC dam.

-Total head values of rectangular mesh have constant difference with PHASE2 software and they have regular variation range.

- Non orthogonal triangular mesh has very low total head variation range in down boundary. Total head variation range is increasing while the calculation is closer to the boundary which is located beneath the dam foundation and the MATLAB and PHASE2 computed heads will be close together.
- Orthogonal triangular mesh has low total head variation range in down boundary but it has more compatibility with reference results than non orthogonal triangular mesh in this section. Total head variation range is increasing while the calculation is closer to the boundary which is located beneath the dam foundation and the MATLAB and PHASE2 computed heads will be very close together.
-The computed P-VALUES are between 0.8-1 that demonstrates high ability of presented mathematical-numerical model in head evaluation beneath the roller compacted concrete dam.
-Finite volume method have so many advantages rather than other numerical methods, finite element, such as suitable compatibility with studied domain specially beneath the roller compacted concrete dam, Easy prescription for seepage equation discretization.
- In spite of utilizing tetrahedron elements in PHASE2 program which is precisely than two or three faced elements, reference and presented model results are very match that demonstrate high accuracy of the mathematical-numerical model.
- Riemann boundary condition is used in order to model the boundary conditions exactly. The reason is that precise head value in some studied domain boundaries is not specified.

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