A Micro-plane Damage Model for Plane Concrete after Exposure to the High Temperature

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Abstract

An attempt is made on a micro-plane damage model to take on constitutive behavior of concrete after exposure to the high temperatures. The constitutive equation using a damage formulation which was developed by Labibzadeh et al. (2006) has been adapted here to account for the effects of elevated temperatures. The damage formulation has been founded upon five fundamental damage functions which are directly related to the loading history of each micro-plane. The available damage functions for the model include a bunch of experimental variables dependant on thermal and mechanical issues. Optimum values for these parameters require meticulous experimental treatments to draw a precise conclusion either on uniaxial or multiaxial responses. Also elastic modulus varying with raising temperature is accepted as a clue to push thermal damage to the model.

The characteristic feature of the proposed model has been investigated by published experimental results for uniaxial compression and tension tests. This model is developed in Visual Fortran computer language and could be easily implemented into a 3D finite element code to present a crack-damage analysis, also understanding the behavior of designing structures during the fire events.

Key words: Constitutive equation; Micro-plane; Damage; Crack analysis

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1. Introduction

Generally, most of concrete structure exposed to fire could be retrieved and returned to service even after severe fires. In this process the damaged structural members must reach to a minimum strength, ductility and stiffness which they have possessed before the fire. After heating concrete to the high temperatures, a series of physical and chemical reactions lead it to exhibit changes such as loss of moisture, decomposition of aggregate particles and dehydration of cement past. These changes could strongly affect the structure of concrete members by reducing mechanical properties of concrete, namely the decrease in both strength and stiffness of the concrete. Also variation of brittleness of the softening behavior is going to show more ductility with raising the temperature. To investigate and rehabilitate the mentioned structural members this point is crucial to have a good estimation on effects of temperature on mechanical properties of concrete. This necessity rises when stress strain relationships are required to predict the entire behavior for a further earthquake.

Many studies have been made to evaluate influence of the high temperature on physical properties of concrete. Consequently numerous models have been proposed to account high temperatures for concrete. Simo (1987) proposed continuous elasto-plastic damage models with considering strain and stress based dual framework. To propose a unified theory, Carol et al. (1994) merged elastic degradation and damage theory. Another model based on plastic viscosity has been conducted by Faria et al. (1998) for massive concrete structures. Wu et al. (1999) tested 44 specimen exposed to the temperatures up to the 600 $^{\circ}C$ and extracted a stress-strain relation which is applicable for heated and unheated concrete. Thermo-chemo constitutive equations based on plasticity could be utilized to consider decohesion and the thermal damage (Ulm et al. (1999)). Also Hydro-thermo-mechanical analysis along with further development to account the damage at high temperatures has been considered for concrete (respectively Bagio et al. (1995) and Gawin et al (1999)). Nechnech et al. (2002) proposed an elasto-plastic damage model for plane concrete subjected to the elevated temperature in that thermal damage has been defined via the variation of elastic modulus with the temperature.

Researches alluded above have been regarded concrete on the basis of plasticity, damage or a combination between both of them. Generally, continuous models are categorized into two main classes: **macroscopic models** which are presented by damage and plasticity theory or a combination between both. And **mesoscopic models** such as multi-laminate or micro-plane models. Through the mesoscopic notion a model presented a damage model for concrete on the basis of "micro-plane theory" which has employed experimentally damage functions for only mechanical loadings (Labibzadeh et al. (2006)). In the present paper the previous model (Labibzadeh et al. (2006)) is further developed to consider the effect of elevated temperatures. The new model has combined the mechanical damage with the thermal one which is defined with variation of elastic modulus.

2.Data and Material

At first, the slip theory developed based on the idea that constitutive behavior of material could be presented by the behavior of specified planes within the material. By applying this theory to account for continuum damage mechanics and cohesive frictional material, for the first time "slip or multi-laminate theory" changed its name to the "micro-plane theory".

Now we are presenting the applied formulation at the model which is a combination of micro-plane theory with assumption of kinematic constraint approach and damage theory. Models constrained with this approach are capable to depict softening behavior of plane concrete in a stable manner. The unite sphere of micro-plane models includes 26 planes tangent to the sphere's surface (Fig. 1). The position and orientation of each plane is specified with the unite normal to the plane with components of $n_i, i = 1,2,3$ (any subscript refers to the components in Cartesian coordinate axis x_i). Also to extract shear components on the micro-planes, we are required to define two extra coordinate directions M, L which represent two orthogonal unite coordinate vectors m_i, l_i respectively. According to the kinematic constraint, at first macroscopic strain tensor is projected on the planes. This projection yields three components of micro-strains along the plane's triplet local directions which one is normal (ε_N) and two of them are tangential $(\varepsilon_M, \varepsilon_L)$ .The following relations depict the projection process in mathematical form:

$$\varepsilon_N = N_{ij}\ \varepsilon_{ij}, Nij = ninj \qquad (1)$$

$$\varepsilon_M = M_{ij}\ \varepsilon_{ij}, M_{ij} = (m_i n_j + m_j n_i)/2 \qquad (2)$$

$$\varepsilon_L = L_{ij}\ \varepsilon_{ij}, L_{ij} = (l_i n_j + l_j n_i)/2 \qquad (3)$$

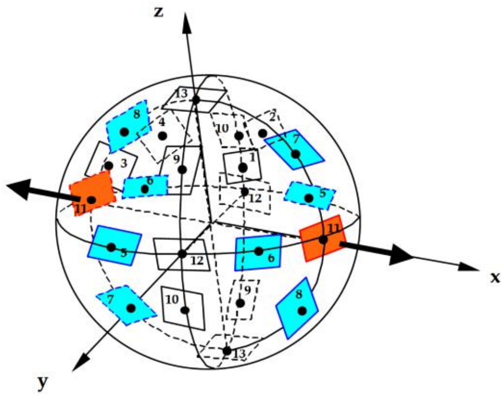

Fig 1. Position of integration points on the unite sphere surface.

Where repeated indices imply summation over $i = 1, 2, 3$. Now with employing the appropriate constitutive laws at micro-plane level, micro-stresses are updatable through the executed strain components on the planes.
Totally, kinematic constraint models indefeasibly oblige the macroscopic stress tensor to have identical projection on the micro-planes with the stress components at the same planes. Thus these models are credible if and only if the constitutive laws at micro-level are particularly invented in a way that this condition could be satisfied. Due to the desired satisfaction through the analysis procedure is generally scared, the kinematically constrained models have been proceeded to fabricate a static equilibrium between plan's stress components and macro-level stress tensor. This equivalence could be supplied by means of the virtual work method. The equation (2) has equated the virtual work inside the unit sphere and on its surface:

$$\sigma_{ij} = \sigma_v \delta_{ij} + \frac{3}{2\pi}\int_{\Omega} [\sigma_D (N_{ij} - \frac{\delta_{ij}}{3}) + \sigma_L L_{ij} + \sigma_M M_{ij}] d\Omega \qquad (4)$$

Where Ω is the unit hemisphere surface, σ_L and σ_M are tangential stress components on each plane, σ_v and σ_D are volumetric and deviatoric parts of normal micro-stress components. The introduced integration at equation (4) can be carried out by any numerical integration technique such as Gaussian integration. Here, an approximate formula with 26 integration points with a finite number of micro-planes for each point has been taken into account. The final portrait of the numerically performed equation (4) is:

$$D_{ijkl} = \frac{3}{4\pi}\int_{\Omega}\left(\frac{E}{1+\nu}\right)[(N_{ij}\ \frac{\delta_{ij}}{3})(N_{kl}\ \frac{\delta_{kl}}{3}) + M_{ij} M_{kl} + L_{ij} L_{kl}] d\Omega + \frac{E}{1-2\nu}\frac{\delta_{kl}}{3}\delta_{ij} \qquad (5)$$

3. Research Methodology

3.1 Elastic modulus at various temperatures

Generally with increasing temperature, concrete structures will be accompanied by changes such as reducing compressive (tensile) strength and increasing peak strain which means concrete is softening with raising temperature. For the same thermal condition, the reduction in compressive (tensile) strength is smaller than its counterpart in the elastic modulus. As shown in Fig. 2 the elastic modulus decreases with increasing temperature. Therefore to assess concrete structures damaged during severe fires, it is crucial to have a precise visualize on the thermal effects over the concrete elastic modulus. For practical purposes at compressive state, the elastic modulus of heated concrete could be considered as secant modulus at 40% of experimental peak strain of compressive stress-strain curve. This percentage for the temperatures 200,400,600 is respectively reported as 80%, 40% and 6%.

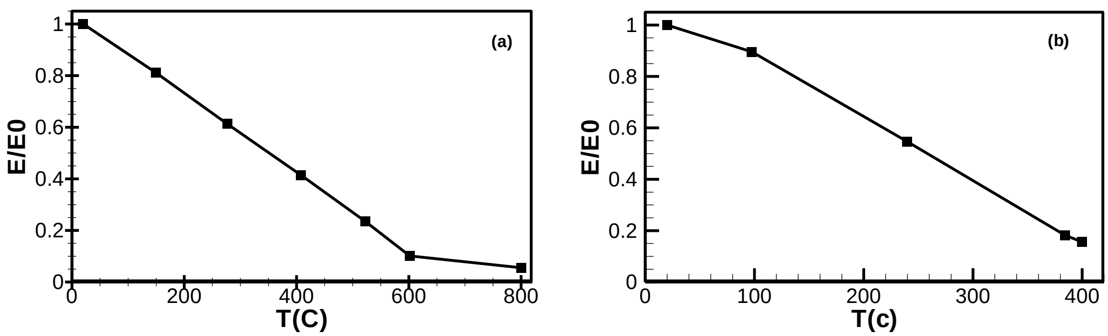

Fig 2 (a, b): Variation of elastic modulus, (a) Compressive state (b) tensile state, with temperature (Nechnech et al. 2002).

3.2 Anisotropic damage model:

Total deviatoric part of constitutive matrices is computed from superposition of its counterparts on the micro-planes that such counterparts in turn, are calculated based on the damage occurred on each plane depending on each specific loading condition (Labibzadeh et al. (2006)). The prime skeleton of damage mechanism at the proposed model is based on five separate damage functions which one by one are constructed to cover one of the alluded loading situations. The five loading states are related as:

1. Hydrostatic compression
2. Hydrostatic extension
3. Pure shear
4. Shear + compression
5. Shear + extension

The damage evolution functions (proposed by Labibzadeh et al. (2006) and here is adapted for thermal consideration) are acquired through the authoritative laboratory tests which are carried on the concrete specimens under diverse compression and tension states of loading. Generally speaking, each function that is formulated for one of the five force conditions has been constructed upon two series of parameter. The first kind (parameter a to k) consists of 11 experimental parameter in a way that each of them has been allotted with considering both proportion of applied macro-level forces at loading directions and thermal condition. These parameters remain constant during loading procedure. Then, they only depend on the ratio of loading at applied directions while in the other hand should be allocated at different temperature ranges. The calibration process for them is a crucial step to satisfy both loading and thermal condition by implementing optimum values. The second type only consists of average strain parameters that are separately concluded from operative micro-strains on the planes. The parts of damage function related to the loading condition 2, 4 are presented here:

$$\begin{cases} \omega_T = 1 - (\dfrac{a}{\varepsilon_{eq}}) \times \exp[-(\dfrac{\varepsilon_{eq} - a}{c - a})] & if\ \varepsilon_{eq} > a \\ \omega_T = 0 & if\ \varepsilon_{eq} \le a \end{cases} \tag{6}$$

$$\begin{cases} \omega_{HT} = 0 & if\ \varepsilon_{eq} \le \sqrt{3}a \\ \omega_{HT} = 1.0 - (\dfrac{\sqrt{3}a}{\varepsilon_{eq}}) \times \exp[-(\dfrac{\varepsilon_{eq} - \sqrt{3}a}{b - \sqrt{3}a})] & if\ \varepsilon_{eq} > \sqrt{3}a \end{cases} \tag{7}$$

3.3 Damage evolution

In our formulation the thermal damage is defined by means of the variation of elastic modulus with temperature. The anisotropic damage model takes advantages of a total damage function ranging from zero to one. Covering the mechanical and thermal effects, the value of zero expresses a micro-plane surface with no crack or any crack initiation and the one is related to a totally damaged surface.

The proposed Total Damage Function (TDF) is assembled for each plane through the aforementioned five damage function and also five new variables (HT, T, SH, C, HC). These new ones are referred to the updated micro-stress components from projected strains on the micro-planes. This could be formulated as:

$$TDF(P) = HT(P) \times \omega_{HT}(P) + T(P) \times \omega_T(P) + SH(P) \times \omega_{SH}(P) + C(P) \times \omega_C(P) + HC(P) \times \omega_{HC}(P) \tag{8}$$

Where P indicates the micro-planes number 1 to 13. Planning procedure for micro-plane stress components employs macroscopic parameters υ, E in which contains thermal effects. Thus the Total Damage Formula is separately computed for each micro-plane on the basis of the variables affected by thermal and mechanical circumstances.

3.4 Model flowchart

Analysis procedure sequence followed by the model has been portrayed at Fig. 3.

4. Results and Analysis

In this section validity of the proposed model has been examined by focusing on its capability to simulate constitutive responses of specimens submitted to the uniaxial compression and tension tests. With this in mind concrete specimens with $f_c' = 27\,Mpa$ is considered.

4.1 Uniaxial compression (UC) test

The uniaxial compression tests on concrete performed by Chang et al. (2006) are conducted here to assess coincidence between simulated responses of the proposed model and experimental stress-strain curves. The test instruction exposes concrete specimen to progressive temperature increase up to the desired level and then this is followed by natural cooling down. Afterward the specimen is conducted to the desired loading to get the stress strain responses. Fig. 4 shows a reasonable agreement between the results fulfilled by the proposed model and experimental stress-strain curves.

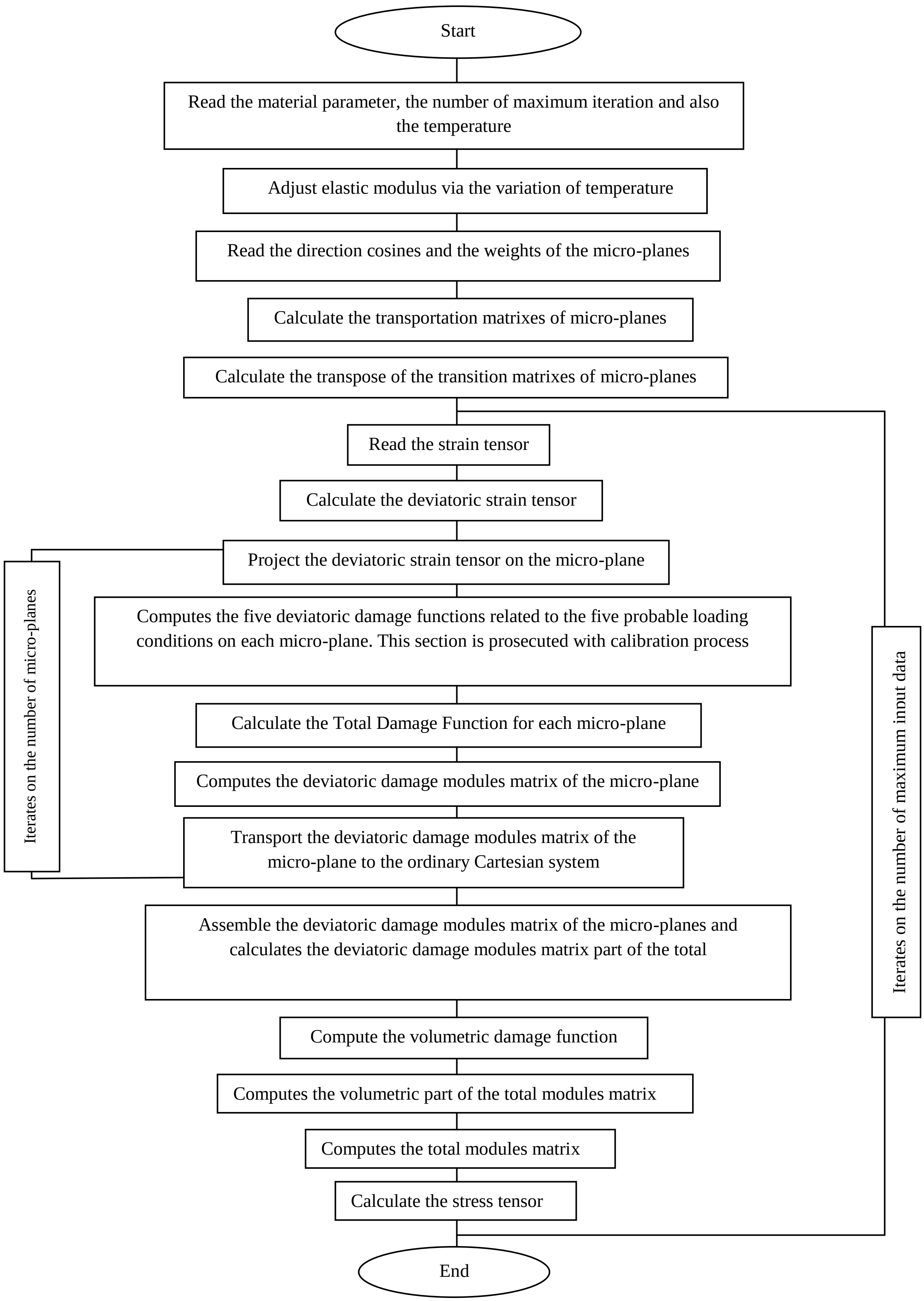

Fig 3: Corresponding model flowchart.

These results declare as temperature increases, both strength and material stiffness decline, concrete is softening and stress-strain curves become flatter. That's why the shape of the curves varies from unheated to the heated states and also between the ones at different temperatures.

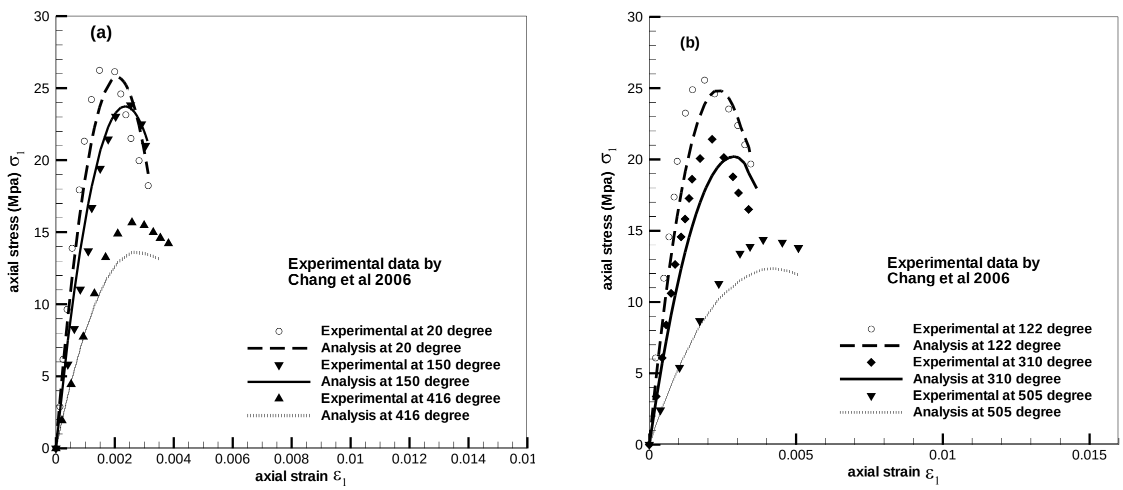

Fig 4: Comparison of proposed stress-strain curves with experimental results at UC test.

Now with applying the uniaxial compression load along the X-axis, the micro-plane 11 on the unite sphere is just loaded by compressive stress. Only tensile stress affects the planes number 9,10,12,13 situated normal to the loading direction and on the other eight micro-planes compression together with shear stresses operate with different ratios depending on geometric location of the plane.

4.2 Uniaxial tension (UT) test:

The tests performed by Felecitti and Gambarova (1998) are considered here. The experimental stress strain responses of concrete specimen under uniaxial tensile loading at different temperatures (reported by the mentioned authors) in compare with simulated results of the proposed model are depicted at Fig. 5(a).

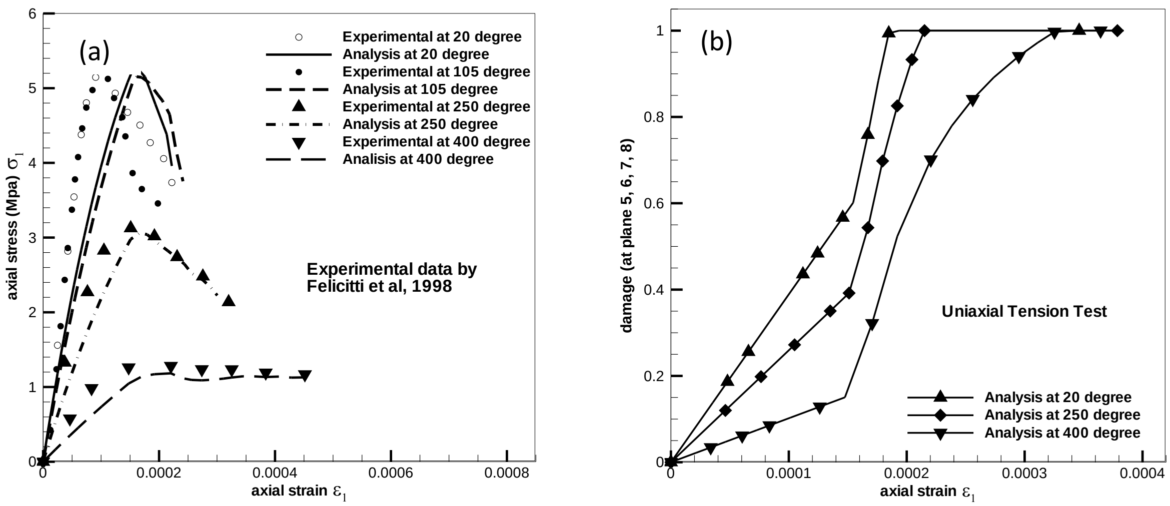

Fig 5: Uniaxial tension results, (a) Axial stress-strain curve, (b) Damage-axial strain curve.

The damage values during the test are presented at Fig. 8. As we can see at the stress-strain curves, together with the raising temperature, the concrete softening behavior conveys the peak location in that its value increases in the strain axis. This event is clearly observable through the gradient of the damage-strain curves (Fig. 5(b)) in a way that with raising temperature the curves become less inclined to absorb higher strains. Furthermore the micro-stress components fulfilled by the projection of macroscopic stress tensor, versus the axial strain are depicted at Figure 6 for the micro-planes number 1 to 4 (the same for all four planes).

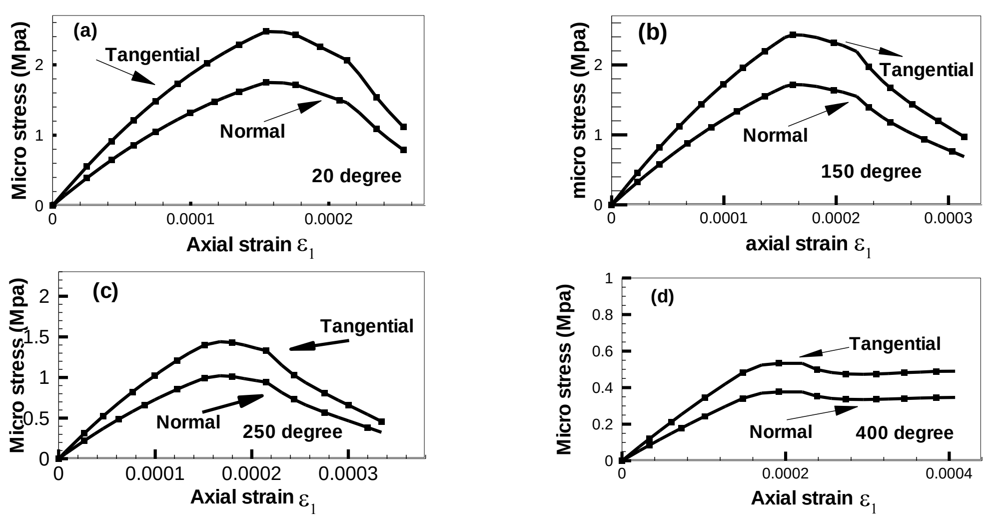

Fig 6: Microstress-strain curves at various temperatures during UT test.

8. Conclusion:

Because of demoting the mechanical properties of concrete such as elastic modulus and compressive (tensile) strength with elevating temperature, the proposed model is defined the thermal damage via the variation of elastic modulus with temperature.

The model implemented five separate damage functions to cover up the total damage included mechanical and thermal ones. The damage functions act via the projected strain components on the micro-planes and the parameters calibrated for thermal and mechanical considerations by published experimental researches.

Raising temperature profoundly affects softening behavior and peak location of the stress-strain curves which the ascending part is turning to the linear state while descending curve becomes flatter.

Macroscopic strain tensor allocates three strain components, projected along the local coordinate axis, to each micro-plane (N- normal direction and M, L- tangential directions). This proficiency advantages the model to predict the direction of crack growth around a point.

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