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Structural Systems According to the Mathematical Theory of Fractals

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Abstract

The fractal structures are highly economical, self-sufficient and rapidly deployable models for universal buildings, giving us an opportunity to develop intelligent and healthy spaces for the masses, even under unfavorable conditions and with limited resources. Owing to the flexibility in design, it can adapt to varying landscapes, site locations, cultures and a vast number of functional applications. The properties of self-similar repetitions, abundance of textural details, and cascades of shape in architecture have been characterized by fractal geometry. Architecture can take advantage of the complexity sciences, by the use of present day computer technology, where algorithms of mathematical and geometric functions can produce new motifs of design. Some contemporary buildings approach the idea of fractal architecture by reintroducing both curvatures and subdivisions at different scales, or a self-similar structure of the same motif, thus producing complexity and formal variety. Any form that is self-similar is likely to be fractal. If there is a regular motif or design, which repeats itself as the structure grows or scales - through time or across space, then the structure can be envisaged as a hierarchy, and thus a fractal organization is a hierarchical organization with applications of point-fixed glazing systems range from simple structures, such as shop windows and shelters, to more complicated ones, such as multi-storey buildings. This paper attempts to propose an approach where structural systems can be developed according to the mathematical theory of fractals.

Key words: Architecture, Contemporary Buildings, Fractal Structures, Hierarchical, Self-similar

1. Introduction

The properties of self-similar repetitions, abundance of textural details, and cascades of shape in architecture have been characterized by fractal geometry. Coined by Mandelbrot in 1970s, fractal theory has been widely applied for the analysis and synthesis of architectural and urban designs. The term, 'fractal,' comes from the Latin word 'Fractus' which means 'broken' or 'irregular' or 'unsmooth'.

Mathematical property for the creation of fractal geometry has been known as iteration. Different rules of iteration create different fractal figures. The property of endlessness in the process of iteration makes fractal figure as the microcosm of infinity. Due to its never ending nature, fractal geometry cannot be possible to explain by Euclidian geometry, but be possible by computer-generated programs and computational techniques such as Shape Grammar. In nature there are plenty of fractal-like objects which are not true fractal, but up to certain range of scales they display fractal properties. Although employed in various fields for different roles, fractal geometry has been applied particularly, in architecture as a language which translates the beauty of complexity as well as the ideas of architects. It also reflects the process of universe and its energy through the buildings.

2. Fractal dimension and box counting method

In fractal geometry, fractal dimension is the quantity which measures the fractalness of an object. For any fractal object fractal dimension is always non-integer, i.e., unlike integer-dimensional objects (zero-dimensional point, one dimensional line, two dimensional square and three dimensional cube) fractal object is a fractional-dimensional which may be in between one dimensional and two dimensional object, or in between two dimensional and three dimensional object. For in between one-dimensional and two-dimensional fractal object fractal dimension 'D' is always more than 1 and less than 2 (i.e., $1<D<2$), whereas for in between two- and three-dimensional fractal object the fractal dimension 'D' is always more than 2 and less than 3 (i.e., $2>D>3$). There are various methods to evaluate the fractal dimension, among which the 'box counting method' is suitable for measuring the fractal dimension of the elevation of buildings, mountains, trees or any objects which are not true fractal. Albeit, 'self-similarity dimension method' is very common for calculating the fractal dimension, but the method is only applicable for the true mathematical fractals where self-similar structures are found at all zooming scales. Fractal dimension calculated by 'box counting method' measures the fractalness of an image on the basis of roughness or textureness or the amount of details. Images having fractal dimensions 1.1–1.5 indicate less roughness and details they have, whereas images having fractal dimension 1.6–1.9 but less than integer value 2, exhibit high textureness and abundance of details. Roughness of the images having fractal dimensions 1.21–1.25 or the dimensions 1.81–1.86 are visually not so much distinguishable by naked eyes. But naked eyes can easily distinguish the images having fractal dimensions 1.1, 1.2, 1.3,…, 1.8 and 1.9, separately. To evaluate the fractal dimension of an image by 'box counting method', firstly a squares grid is overlaid on the image where the size of each grid box, let 'S', determines the scale of the grid. Then the boxes, having any mark or line of the image within the grid, are counted; let it is 'N'. After repeating the same process on the same image by changing the box size, fractal dimension 'D' of that image can be obtained by transforming the results of 'S' and 'N' into the log–log graph. The slope of the resulting line of the log–log graph determines the fractal dimension of the image. There are various software that can directly calculate the fractal dimension of an image by following the basic process of 'box counting method'. In this paper, the software of 'Box Counting Demonstration'17 has been employed for counting the marked boxes. For the analyses, all parts of the temple have been taken as two-dimensional images by keeping the

information about image sizes and their resolutions. The decline of using the fractal geometry in modern architecture became soon eradicated by the strong affection towards the chaos and complexity in contemporary architecture. For searching the new, complex, fractal and chaotic forms in the contemporary Architecture, on the one hand, manual experiments of architectural elements with the fractal geometry are extensively practiced. On the other hand, by adopting the theories of folds, fractals, chaos, complexity and algorithms, 'computer architecture' has been developed that translates the theories into the architecture .Accordingly, the fractal geometry not only analyzes the old buildings and the existed urban patterns and growth, but also helps to create a countless unimaginable forms and patterns for finding new possibilities of space as well as aesthetic in contemporary architecture.

This paper aims to translate virtual fractal models into physically built architectural objects. The considered fractal models are based on iterative algorithms, which were developed at the LIRIS for the creation of virtual images. The physical objects will aim for an application in the field of construction on the scale of architecture and design objects. Fractal objects will be designed and built as bearing shell structures, irregular three dimensional polygonal structures etc.

Generally, since the eighties, architects show a great interest in complex geometry. In the field of construction, until today, it is technically very difficult to build complex shapes and geometries. Therefore, these types of shape have only been applied in the field of industrial design. Contemporary architecture uses CAD software (Computer Aided Design) that enables architects to easily create complex shapes. The complex shapes, proposed by actual CAD-software, obey to a classical approach of geometry because they are locally plane or smooth. In order to build so called 'free-form' shapes, their generally polynomial geometry first has to be discretized, which leads to a more or less approximate result. Passing by the method of fractal geometry, the project presented here tries to face this problem by proposing a set of powerful tools in order to physically construct complex geometries.

Defining shapes by generative systems based on iterative construction presents an advance beyond the limits of classical algebra-differential models. Among the generative systems, subdividing schemes permit the easy generation of surfaces. Based on this method, the project team plans to create a class of fractal objects for CAD-modelers, which can be used for the conception of realizable physical objects. This new class allows the creation of objects of unusual shape (rough, accidental, etc.). Surfaces representing natural shapes (mountains, rocks) as well as surfaces of special shapes (sculptures, architecture, decoration) will be integrated in the CAD modeler.

The advantages to use discrete fractal modeling for conception and production of architectural objects are the following: First, the virtually conceived model consists of a discrete number of elements right from the start (vertex, edge, face). Second, without converting the object, the applied method allows to verify the geometry on physical models at an early design stage by using rapid prototyping. Third, numerical structural simulation can be applied directly on the object's elements. Fourth, for the construction of material and real scale buildings, the geometric elements are replaced by constructive and architectural elements (joints, beams, panels). During this project, different applications will be considered and tested by the construction of several experimental and real scale prototypes. Their applications can be classified into two categories: load-bearing structures (shell-structure, framework, treelike pillar) and non- bearing structures (spatial separator, suspended ceiling, shading panels, décor).

3. Fractal geometry and modeling

Since about 1970, fractal geometry as a new branch of mathematics influences the development of science in various fields (physics, chemistry, medicine etc.). In addition, its mathematical methods influenced the visualization software. Here is a summary of some domains in which contemporary fractal geometry is applied:

-3D virtual imaging (mountains, plants, clouds, etc.)

-physics, chemistry, physiology

-digital image compression (JPEG)

-technologies (fractal antennas)

Apart from this, to our best knowledge, no application of fractal geometry exists covering the overall construction process. Consequently, this project opens a new and unexplored research field. Nonetheless, we will briefly discuss some former works dealing with fractal geometry.

4. Fractal geometry and architecture

The use of fractals in architecture at the scale of a city has been studied by HUMPERT and BATTY who analyze the city by means of different fractal aspects, which are: self-similarity in scale (of the city's texture), fractal growth (looking at the overall shape in different times) and fractal size distribution (dispersion and aggregation of urban agglomerations).

The construction of architectural shapes generated by algorithms based on fractal geometry is a new research field without any scientific paper as far as known.

Starting on a well known fractal model, the IFS (Iterative Function System), it is possible to define a formalism based on systems of subdivision matrices. Under certain conditions, this formalism allows one to obtain shapes having a given topological structure (wire frame figures, surface figures or solid figures), which aspects can be modulated (regular or irregular, smooth or rough). Shapes with attractors (curves or surfaces), currently used by actual modelers are particularly 'smooth'. These fractal models describe parametric shapes that are mathematically well defined and point by point computable. A specific approximation method for curves and surfaces allows the reconstitution of rough objects.

5. Experimental applications

The following kinds of prototypes are planned to be built physically in real scale:

5.1. Fractal Wire Frame Structure

A fractal skeleton framework, which is constituted by tree-like pillars, incorporates a certain kind of fractal notion. The classical system of construction works with several fixed scales: primary structure, secondary structure, tertiary structure, etc. In its bifurcation, a treelike pillar contains these scales at different steps of iteration. L-systems and, more generally, IFS describe apparently highly complex structures with some simple basic rules. In the field of construction, different research on tree like pillar has been done. FREI OTTO's work [8] has also led to interesting treelike constructions. The reflection behind was different though. FREI OTTO observed natural phenomena in order to optimise and lighten his designs. This led partially to an imitation of natural shapes. Contrary to this, our research is interested in the fractal formalism in order to geometrically describe these complex shapes(Fig. 1).
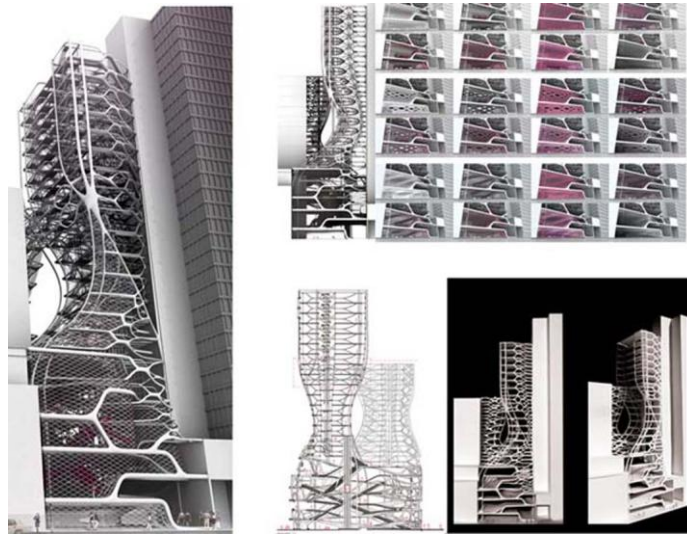
Fig 1 : Momo Tower Project, LA: steven ma, tri do, sanne kampinga

5.2. Fractal Shell Structure

The study aims at the definition of bearing structures obeying the law of fractal geometry. The function of these shell structures is to define interiors, sovereign shelters or dome Structures . The two following cases will be examined: First, a series of shells constructed by IFS will be built. This will generate a family of surfaces which are fractioned into self-similar parts. Second, given (algebra differential) surfaces will be expressed by a fractal formalism. This avoids forcing these 'free-form' geometries to be fitted into a classical mesh (based on regular triangles or regular squares) rather than fitting/fractioning the mesh onto the topology of the initial geometry. Structural calculation methods, which include the fractal notion of this mesh, will be developed (Figs. 2~5)
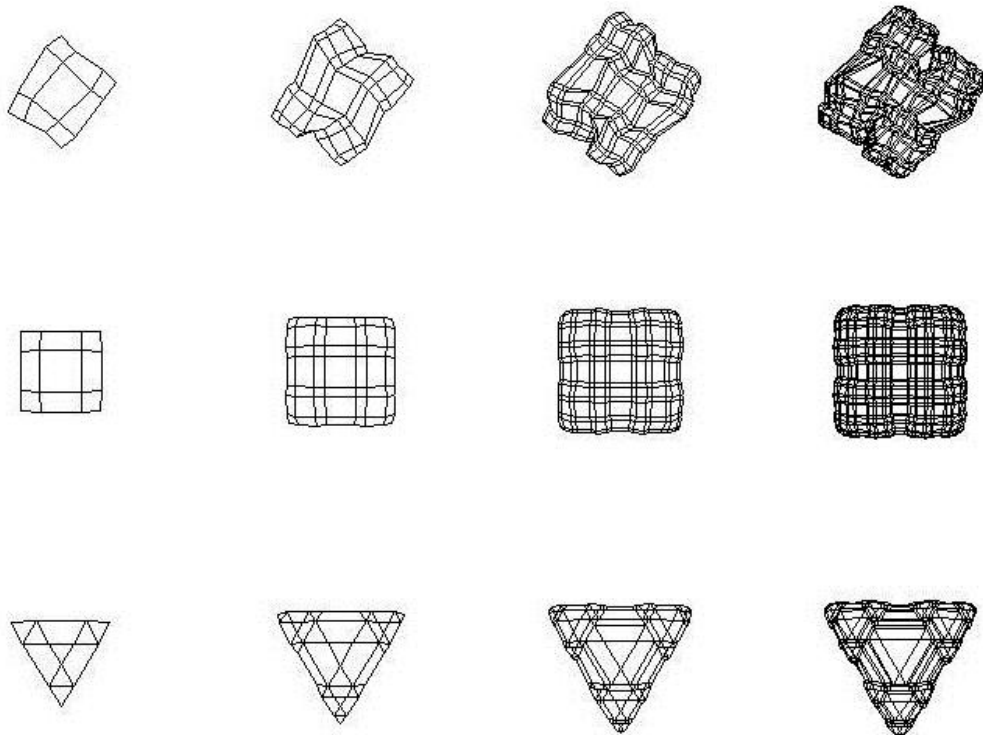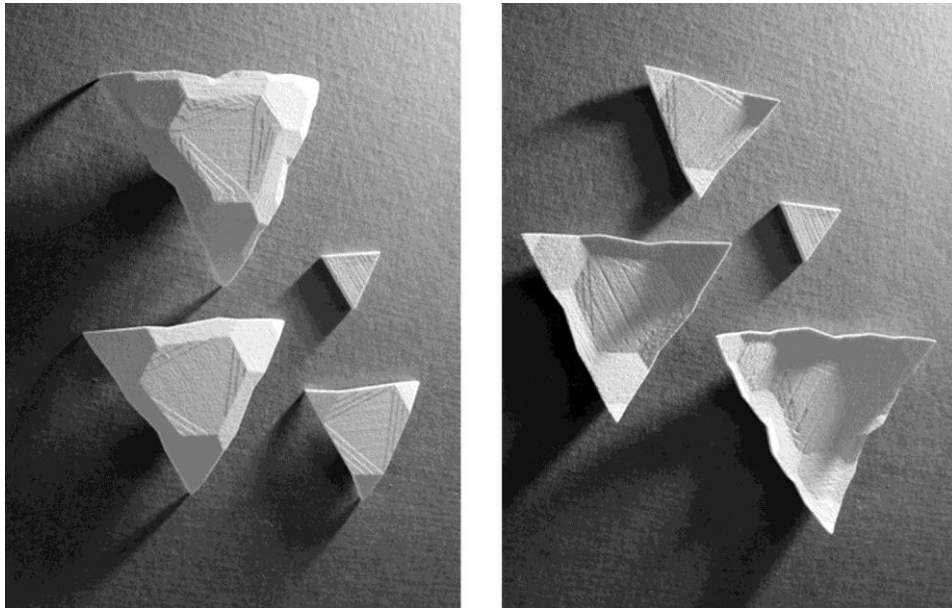
Fig 2 : Fractal Shells and their expanding

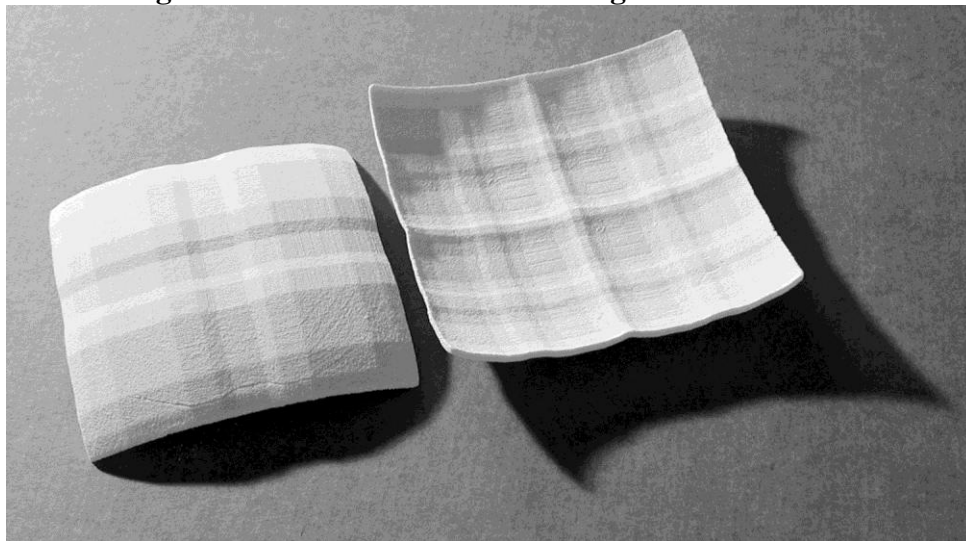**Fig 2 :** Fractal Shells   **Fig 3 :** Fractal Shells

Fig 4 : Fractal Shells

5.3. Fractal shading panel

Fractal curves, which are efficient in terms of filtering/dosing the light but also in terms of its decorative aspect will be applied to mobile or fixed shading panels; blinds likely to arabesques. These objects have a rather ornamental and pleasant dimension. The blinds issued by the consideration of fractal geometry are regarded from the point of view of comfort. Its shading qualities could be topologically compared to the shadow given by a tree. The blinds are inspired by nature (sic) and therefore approach natural conditions. They contribute to a more diversified and richer environment than the one provided by existing shading elements.
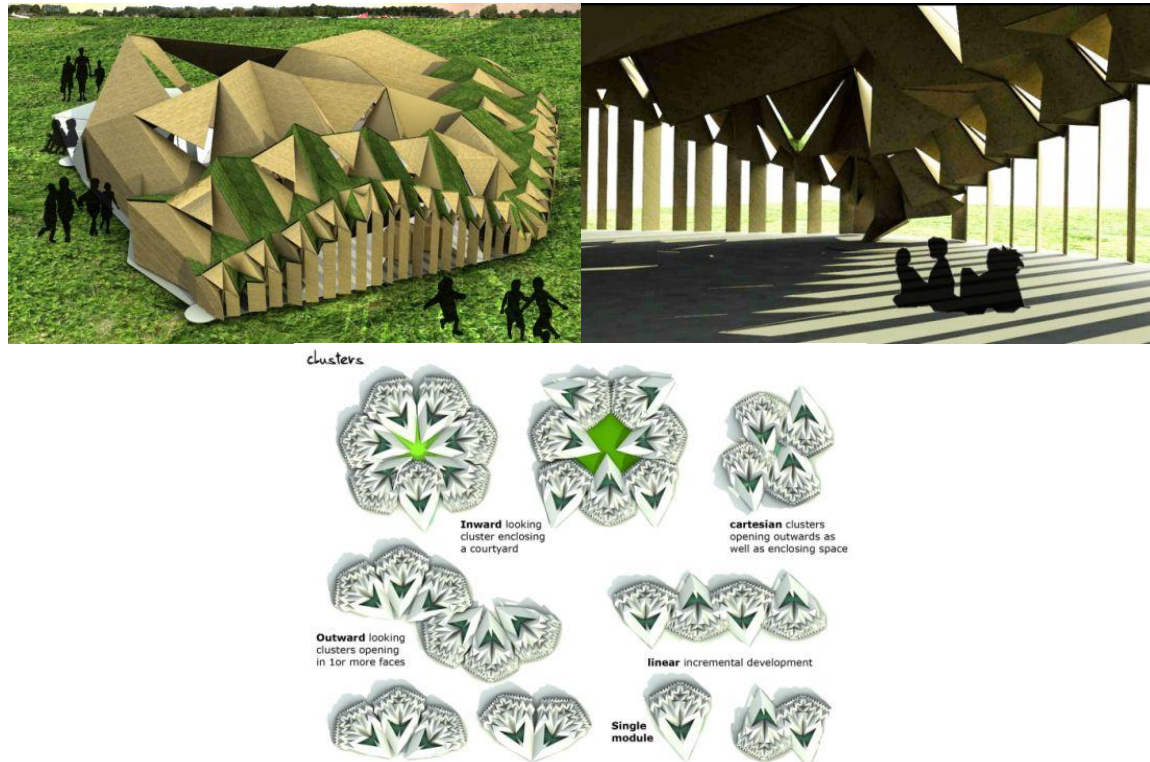
Fig 5 : Economical, self-sufficient and rapidly deployable models for universal buildings.

6. Conclusions

Nowadays, centre–periphery structures seem to dominate the region. Peri-urbanisation has affected most communes and traditional village structures (compact, ribbon ,etc.) are rare, as they are progressively transformed into more uniform clusters of buildings which loose the tentacular aspect often observed in the first phase of urbanisation. Indices of fractal dimension represent the morphology of the built-up environment as well as its extent. They are – on average quite independent of the appearance of the built environment. Hence suburban landscapes are characterised by a wide variety of land uses, creating complex and diverse landscapes consisting of a highly fragmented mosaic of different forms of land cover and a dense transport infrastructure.

The fractal dimension measured on surfaces gives different information from that measured on borders. Fractal dimensions are of value in studying built-up landscapes as they provide a theoretical basis for the observed features of self-similarity and scale-independence. They also provide a quantitative measure of the roughness or complexity that could help in the interpretation of maps of built-up areas, and assist the explanation of processes creating built-up landscapes. Linking surface dimensions and perimeter dimensions revealed counterintuitive results, for which a heuristic explanation has been found and verified. Nevertheless, more multidisciplinary research on other regions is required to decide whether the relationships observed here reveal fractal properties of the shapes or whether they are an artefact. Fractal measures serve to characterise the spatial organization of urban patterns with unequivocal values. They can be used to measure to what extent the built-up area is distributed in a uniform or a varied way within an urban pattern. When interpreting the results it becomes obvious that a fractal approach to urban patterns helps to improve our knowledge of their spatial organisation, regardless of the extent to which they were planned. Obviously multi-scale pattern organisation is an interesting way of managing the consequences of the new peripheral lifestyle which tends

to have good access to different kinds of urban and rural amenities, while simultaneously reducing the risks of a diffuse sprawl which tends to reduce the quality of the environment and generate more and more traffic flow. This empirical work can also help to inform the choices made in urban simulations.

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