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Wavelet for Estimation of Fractal Dimension in ALOS-PALSAR Images

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Abstract

Fractional Brownian Motion (fBm) has been successfully exploited to model an important number of physical phenomena and non-stationary processes such as remote sensing image. These mathematical models closely describe essential properties of natural phenomena, such as self similarity, scale invariance and fractal dimension. There are several methods to estimate fractal dimension in Fractional Brownian motion model. The use of wavelet analysis combined with fBm analysis may be provide an interesting approach to compute key value for fBm Processes, such as fractal dimension. In this paper we used power spectrum approach to calculate the Hurst coefficient (H) and then fractal dimension for both one-dimensional and two-dimensional signals and then tested the algorithm, on ALOS-PALSAR (Japanese satellite) image. Fractal dimension of image indicates the edge detection algorithm so we compared the results with classical edge detection method such as canny and sobel edge detector.

Key words: Fractal Dimension, Remote Sensing Images, Wavelet Multiresolution

1. Introduction

Fractal geometry was introduced and popularized by Mandelbrot to describe highly complex forms that are characteristic of natural phenomena such as coastlines and landscapes. The main attraction of fractal geometry stems from its ability to describe the irregular or fragmented shape of natural features as well as other complex objects that traditional Euclidean geometry fails to analyses. Although the fractal images are the popularized class of fractals, there are also numerous natural processes described by time series measurements (e.g., noises with power spectrum; econometric and demographic data and pitch variations in musical signals) that are fractals. The relevance and usefulness of fractal geometry to solving remote sensing problems can be attributed to the fact that remotely sensed images are not only spectrally and spatially complex, but they often exhibit certain similarities at different spatial scales (SUN et al, 2006). Fractal models have been used in a variety of image processing and pattern recognition applications. For example, Tzeng and Chen (2006) have applied fractal techniques in change detection and Kumar (2008) used fractal dimension in reduction of hyperspectral images. Applications of fractal techniques to image analysis rely heavily on the estimation of fractal dimensions. The fractal dimension is a key parameter developed in fractal geometry to measure the irregularity of complex objects. There are many different approaches to compute fractal dimension. These algorithms generally fall into three classes. The first approach is derived from the point set topology of fractals; according to this

technique, referred to as the mass scaling technique, one measures the mass of a fractal surface within a resolution cell of size L (denoted by M(L)), A second approach, namely the variance scaling method, relies on the mean square increments. The third approach that used in the present work, derives from the peculiar form of the power spectrum of fBm; since fBm is not a stationary process, fractional Brownian motion does not have a power spectrum defined in the classical sense. Nevertheless, it has been shown that fBm, being an isotropic random field, can be characterized by a random phase Fourier description obeying a generalized power density (Betti et al, 1997). Stewart et al. (1993) have been shown the power spectrum technique seems to yield very accurate estimates and for this reason we explain this method for computing fractal dimension of remote sensing image in the paper.
By developing multiresolution analysis as well as wavelet analysis, the texture analysis of remote sensing images described through fBm was greatly enhanced. The statistical, spectral and properties of the fBm's can be exploited to estimate fractal parameters analyze and so model and measure the texture content of a specific image. This joint study of fractals and wavelets has lead to the development of methods and models to analyze fractional power law processes (Parra et al, 2003). An important contribution in the formulation of a 2-D model is proposed by Heneghan (1996), who describes both the spectral properties and correlation function of an fBm, and proposes a method to estimate the fractal dimension using the statistical properties of the Continuous Wavelet Transform (CWT) of an fBm. Wornell (1993) gives a detailed demonstration on how 1/w processes can be optimally represented in terms of orthonormal wavelet bases, which opens the possibility to the use of discrete wavelet transforms in the present article.
This paper is organized as follows: in Section 2, we first review the concept of fBm process; then, the power spectrum method in computing fractal dimension is discussed; in Section 3, we will the type of data and experimental results and finally there is a summery of research in Section 4.

2. Research Methodology

2.1 Fractional Brownian Motion

Fractional Brownian motion is a Gaussian zero-mean and non-stationary stochastic process $B_H(t)$, indexed by a single scalar parameter $0<H<1$, and is defined as Eq. 1.

$$B_H(t) - B_H(0) = \frac{1}{\Gamma(H+0.5)}\left\{\int_{-\infty}^{0}\left[(t-s)^{H-0.5} - (-s)^{H-0.5}\right]dB(s) + \int_{0}^{\infty}(t-s)^{H-0.5}\,dB(s)\right\} \qquad (1)$$

In Eq. 1 H is the Hurst index. For H=0.5, fBm is an ordinary Brownian motion.
Although fBm is a non-stationary process, it has stationary increment property (Yazdi and Mahyari, 2010).

2.2 Fractal Dimension by Power Spectrum method

We estimate fractal dimension of remote sensing image in global and local form. In global fractal dimension for any gray scale image, compute one fractal dimension that this dimension is between 2 and 3. Global fractal dimension used in urban growth planning for example Shen (2002) compute fractal dimensions of 20 large US cities along with their surrounding urbanized areas. The result shows that larger cities have the bigger global fractal dimension. In most of applications, the estimation of a global fractal dimension from the whole image is not sufficient. In particular, in texture segmentation, where the goal is to segment regions with different textures, it is necessary to estimate a local fractal dimension (LFD). The localisation is generally obtained by applying the estimation on a window centered around each point.

Some of the most frequently seen structures in fractal geometry, generally known as l/w processes, show a power spectrum obeying the power law relationship as Eq. 2.

$$S(w) \propto \frac{K}{|w|^{\beta}} \quad , \quad S(w) = |\alpha|^{\beta} \ S_x(aw) \tag{2}$$

In this equation w corresponds to the spatial frequency, and β= 2H + 1. This kind of spectrum is associated to statistical properties that are reflected in a scaling behaviour (self similarity), in which the process is statistically invariant to dilations or contractions, H in β= 2H + 1 is Hurst index with the restriction 0< H <1. The function that relates fractal dimension (FD) to H is defined as Eq. 3.

$$FD = E + 1 - H \tag{3}$$

where E is the Euclidean dimension.

A. 1-D Case

Spectrum of an fBm follows the power law of fractional order shown in Eq. 3, using either a time frequency description or a scale time description (Parra et al, 2003). MALLAT (1989) showed that if the frequency signal S(ω) is filtered with a wavelet filter ψ(u), the resulting spectrum at the specific resolution is

$$S_{2^j}(w) = S(w)\,|\,\Psi(2^{-j} w)\,|^2 \tag{4}$$

And ψ is wavelet filter. Using the sampling for the discrete detail description of a function f

$$< f(u), \Psi_{2^j}(u - 2^{-j} n) >=)(f * (u) * \Psi(-u))2^{-j} n \tag{5}$$

Or

$$D_{2^j} = ((f(u) * \Psi(-u))(2^{-j} n)) \tag{6}$$

In Eq. 5, < > denote for inner product. D_2^j contains the coefficients of the high frequency details of the function, the spectrum of the discrete detail signal can be written as Eq. 7 (Parra et al, 2003).

$$S^d{}_{2^j}(w) = 2^j \sum_{k=-\infty}^{\infty} S_{2^j}(w + 2^j 2k\pi) \tag{7}$$

By integrating such equations at two different resolutions, the following relationship is obtained:

$$\sigma^2{}_{2^j} = 2^{2H} \sigma^2{}_{2^{j+1}} \tag{8}$$

where σ is the energy of the detail signal. The above result does not depend on the specific resolutions. A linear least square fit performed on Eq. (8) yields:

$$H = \frac{1}{(J+1)\log 4} . \log \frac{\prod_{j=-1}^{J-1} \sigma_{2^{j+1}}}{\prod_{j=-1}^{J-1} \sigma_{2^j}} \tag{9}$$

B. 2-D Case

Extending the analysis to the two-dimensional case, the same steps of the previous section are followed. For a 2-D fBm, the power spectral density assumes the form (Parra et al, 2003):

$$S(w_1, w_2) \propto \frac{1}{|w_1{}^2 + w_2{}^2|^{\beta}} \tag{10}$$

For this part of study, S (w) is computed as the power spectral density of stationary signal, which doesn't apply strictly to non-stationary processes, as is the case. However, this approach leads to good results.

$$S(w_1, w_2) = FFT\,|\,\mathrm{Im}age\,|^2 \tag{11}$$

If the frequency domain signal is filtered with a wavelet filter, the resulting spectrum at the specific resolution is

$$S_{2^j}(\vec{w}) = S(\vec{w})\,|\,\Psi_{2^j}{}^3(2^{-j}(w_1, w_2))\,|^2 \tag{12}$$

Where

$$|\Psi(w_1, w_2)|^2 = |\Psi(w_1)|^2 |\Psi(w_2)|^2 \tag{13}$$

Thus the power spectrum of discrete signal can be put in the form of Eq. (14)

$$S^d{}_{2^j}(w_1, w_2) = 2^j \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} S_{2^j}(w_1 + 2^j 2k\pi, w_2 + 2^j 2k\pi) \tag{14}$$

The energy of the details function at an specific resolution j can be calculated by integration in the support of $\psi_j^3(\omega)$ of the chosen wavelet filter, as shown in Fig. 1. By integrating such equations at two different resolutions, the Eq. (8) is obtained and a linear least square fit performed on it Eq. (9) yields. In following, we reveal steps of the power spectrum density algorithm for computing Hurst index of images:

I. Compute 2-D FFT of image
II. Compute power spectrum density according to Eq. (11)
III. For j=1:N (Nis level of multi-resolution)
    1. Compute wavelet transformation
    2. Sum all the elements of the detail matrix correspond to $\psi_j^3(\omega)$
    3. Divide the resulting matrix by $2\pi 2^j$ to obtain energy at the specific resolution (j).
IV. Estimate H according to Eq. (9)

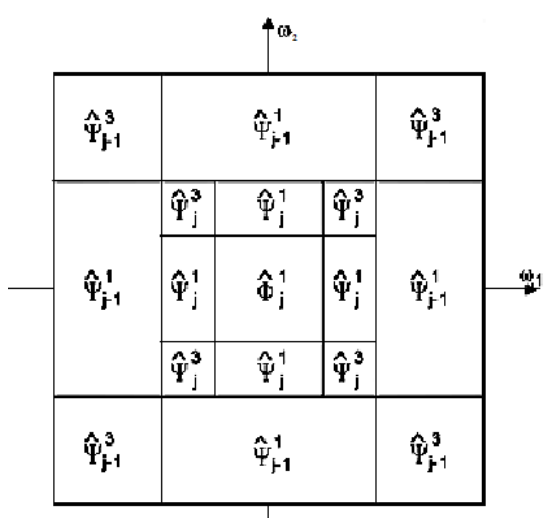

Fig 1: Dyadic frequency allocation for each wavelet filter in a Multiresolution decomposition.

3. Experimental Results

In this section, experiments are performed on the ALOS-PALSAR sample image (Fig. 2-A) by using the power spectrum for computing LFD or fractal image. We used the wavelet Daubechies6 in this paper. In computing local fractal dimension, optimum selecting of window size is issue, because the results are affected by the window size. We implement algorithm with several windows like 7×7, 9×9, 11×11 and 13×13. Global fractal dimension for this image is 2.74 and the results of implemented algorithm with different window sizes (fractal images) are shown in Fig 2-B~2-E.

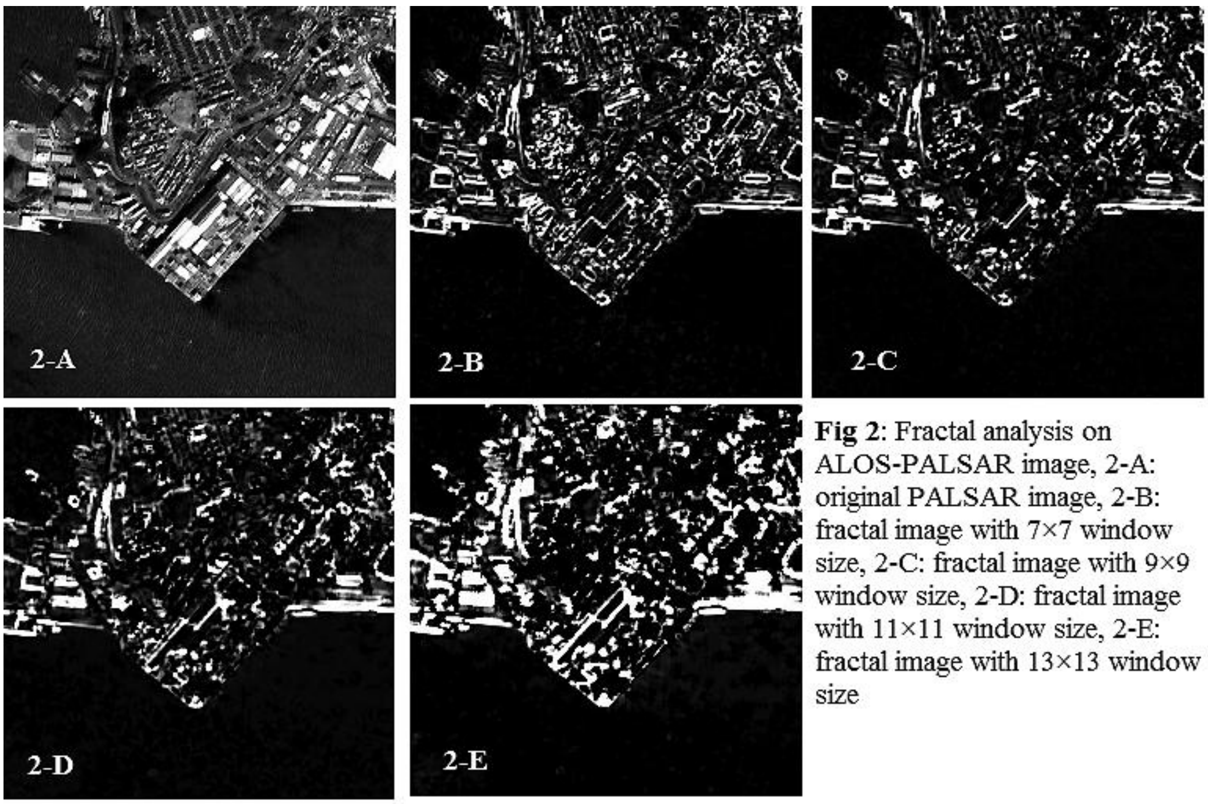

Fig 2: Fractal analysis on ALOS-PALSAR image, 2-A: original PALSAR image, 2-B: fractal image with 7×7 window size, 2-C: fractal image with 9×9 window size, 2-D: fractal image with 11×11 window size, 2-E: fractal image with 13×13 window size

Fractal dimension of image indicates the edge detection algorithm (Klaus and et al. ,1994), so to performance of the algorithm, we used the validation methods of edge detection algorithm. Commonly methods for evaluation are the root-mean-square (e_{RMS}), root-mean-square signal-to-noise ratio (SNR_{RMS}) and the peak signal-to-noise ratio (SNR_{PEAK}) as in Eqs. 15~17 (Roushdy, 2006).

$$e_{RMS} = \sqrt{\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}| O(x,y) - f(x,y)|^2} \tag{15}$$

$$SNR_{RMS} = \sqrt{\frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}| O(x,y)|^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}| O(x,y) - f(x,y)|^2}} \tag{16}$$

$$SNR_{PEAK} = 10 Log_{10} \frac{(L-1)^2}{\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}|O(x,y)-f(x,y)|^2} \quad (17)$$

In this equations O is edge detected image, f is original image and L is number of gray level of used image. The result of comparison with different window size is give in Table 1.

Window size	Evaluation		
	e_{RMS}	SNR_{RMS}	SNR_{PEAK}
Fractal dimension 7 ×7	39.25	2.54	16.25
Fractal dimension 9 ×9	35.77	3.63	17.06
Fractal dimension 11 ×11	34.64	3.51	17.33
Fractal dimension 13×13	48.88	3.13	14.34

Table 1. Comparison results of fractal image with different window sizes

For reveal the high performance of fractal analysis in edge detection we compare the results of 11×11 window size with classical edge detection algorithm such as sobel and canny filter. Canny and sobel edge map are shown in Fig 3-A and 3-B. Table 2 show the results of comparison between fractal and classic methods.

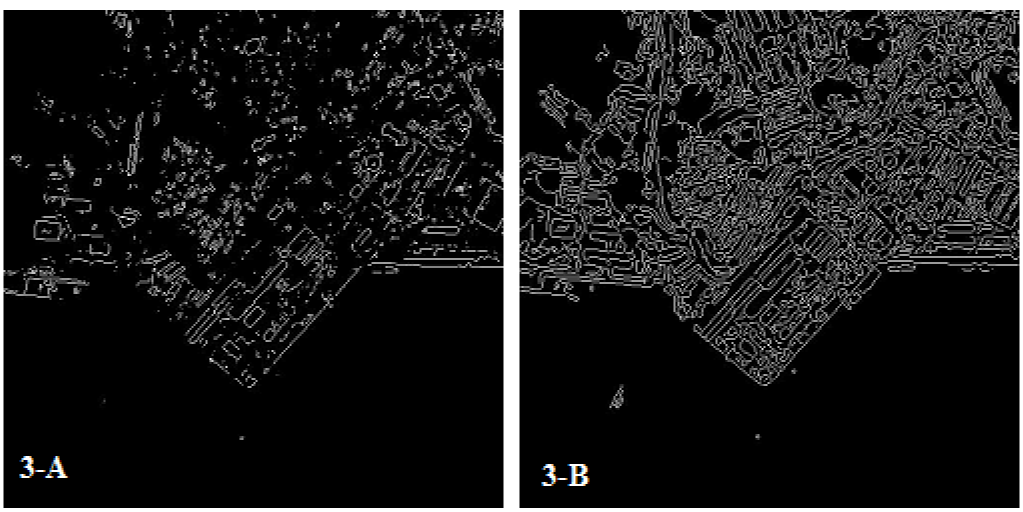

Fig 3. Edge detection with classical method, 3-A: sobel filter, 3-B: canny filter

Algorithm	Evaluation		
	e_{RMS}	SNR_{RMS}	SNR_{PEAK}
Sobel edge detector	126.39	0.0014	6.09
Canny edge detector	126.34	0.0023	6.10
Fractal dimension 11 ×11	34.64	3.51	14.34

Table 2. Comparison results of edge detection algorithm

As can be seen in Table 2, fractal dimension method has a lower MSE and higher $SNAR_{PEAK}$ and SNR_{RMS} than to classical method. It is seem that the fractal analysis have high performance than classical method in edge detection. The classical method can not availably detect the image edge while the image contain noises but in the fractal dimension which indicates that the edge detection algorithm can suppress noises availably and keep the edge details better.
One issue in computing fractal dimension is determining the optimum window size, different sizes of window result the different edge map. It is better to use the different window size in different area of image for example a small window is suitable for expressing the strong edges and it is more appropriate to use a larger window in the smooth surfaces such as a sea. As such, how to choose an appropriate window size in different area of image for computing fractal dimension values is an issue that deserves further research.

4. Conclusions

Fractal geometry appears to provide a useful tool for characterizing textural features in remotely sensed images because most of what we measure in remote sensing such as boundaries of land covers, patches of landscapes, rivers and water bodies, tree crowns, etc. is discontinuous, complex, and fragmented. The fractal dimension is a key parameter developed in fractal geometry to measure the irregularity of complex objects. Fractal dimension in remote sensing image is determined in global and local form. In global form, for whole image or every band one fractal dimension is determined and global fractal dimension is between 2 and 3. Local fractal dimension computed by considering a $w \times w$ window around each pixel in the image. Local fractal dimension have many application in remote sensing such as image classification, segmentation, change detection, edge detection, band reduction in hyper, compression and etc. And also global fractal dimension is used in urban expansion, analysis of remote sensing image for detect noisy bands. There are several methods for computing fractal dimension. Among this, we chose the power spectrum density method for computing Hurst index, both for one and two dimension cases because it seems to yield very accurate estimates. Fractal dimension of image indicates the edge detection algorithm therefore we compared the results of fractal analysis with classical edge detection algorithm. The classical method can not detect the edge while the image contain noises however in fractal analysis, noises suppress and keep the edge details better. In remote sensing image, it is necessary to use the methods that they being ineffective to noise, because almost they have a noise and specifically in SAR images that they have speckle noise. Window size is premier parameter in fractal dimension. Different size of window make different edge map. It is difficult computing optimum size of window in different area of image because we would like use the large window in smooth area and small window in the edge.

A potentially more challenging issue in computing local fractal dimension values is the choice of window size. On one hand, minimization of local window size is required to capture details of local variations in land covers. Use of large window size may, however, lead to several problems. First, a large window will include more land covers and it can lead to mixed pixel problems. Second, using a large window means that land cover features smaller than the window size will not be identified in classification. Third, using a large window will also lead to loss of more pixels on the edges (i.e. boundary effects). As such, how to choose an appropriate window size for computing LFD values is an issue and use of fuzzy logic may be appropriate way in solving it.

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