Analysis of Pore Pressure and Stresses around a Borehole: A Local Thermal non-Equilibrium Approach

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Paper Reference Number: 0112-495
Name of the Presenter: Hamid Roshan

Abstract

The conductive-convective thermo-poroelastic theory has been successfully applied to the problems of geotechnical engineering, in particular wellbore stability in the area of drilling industry. The assumption of "*instantaneous local thermal equilibrium*" has been employed to derive the governing equations for such problems. This assumption, however, falls short in describing the heat transfer in high permeable rocks. The aim of this study is to replace the assumption of "*instantaneous local thermal equilibrium*" with the real assumption of "local thermal non-equilibrium" to solve the conductive-convective thermo-poroelasticity. To do this, a finite element based model of conductive-convective thermo-poroelasticity is developed which takes into account the later assumption to capture the real change in rock's temperature and resulted stresses. From the results of this study it was revealed that there is a significant difference in heat transfer between the classical theory (instantaneous local thermal equilibrium) and LTNE theory (local thermal non-equilibrium). The effective stresses are remarkably affected by this difference in heat transfer. Consequently there is an increase in effective tangential stress around the wellbore whereas a decrease in effective radial stress when LTNE is considered. The results also showed that the change in effective tangential stress is more pronounced than effective radial stress.

Key words: Local thermal non-equilibrium, Borehole stability, Instantaneous local thermal equilibrium, Finite element method, Thermo-poroelasticity

1. Introduction

Wellbore stability problems are often encountered when drilling in relatively high permeable formations. Analysis of stresses plays a significant role in wellbore stability prediction. There are four major mechanisms that affect the state of stresses in the formations around the wellbore namely mechanical, thermal, hydrological and chemical effects. Chemical effect is widely referred to low permeable shale formations (Nguyen and Abousleiman 2009; Roshan and Rahman 2010; Roshan and Rahman 2010). Thus for a rock without the chemical effect, thermo-poroelastic models are often used. A thermoelastic theory of fluid saturated porous media, which is derived based on the difference between thermal expansibility of the rock matrix and pore fluid, was first presented by McTigue (1986) and Kurashige (1989). Based on their works several authors studied the effect of thermal stress and pore pressure on wellbore stability (e.g. Wang and Papamichos 1994; Li, Cui et al. 1998). In all the aforementioned studies the authors showed that the pore pressure and stresses are significantly influenced around the wellbore by change in temperature. Later on, the convective-conductive thermo-poroelastic models were presented where the effect of convective heat transfer on state of stresses and pore pressure around the wellbore in relatively high permeable rocks was studied. (Gatmiri and Delage 1997; Wang and Dusseault 2003; Chen and Ewy 2005; Farahani, Yu et al. 2006; Sheng, Liu et al. 2008). The assumption of instantaneous local thermal equilibrium between rock and fluid, however, was used in all abovementioned studies. This assumption seems to be the source of remarkable inaccuracy where stress analysis is needed in high permeable formations.

Meanwhile, several authors studied the effect of both LTNE and classical theories on only heat transfer by forced convection and steady state flow condition (Nield, Kuznetsov et al. 2002; Rees, Pop et al. 2005; Celli, Rees et al. 2010). The other authors also investigated the thermo-poroelasticity considering both theories, however, it is limited to thermal conduction only (He and Jin 2010).

In this study, the convective-conductive thermo-poroelastic model is developed which takes into account the transient change in convection-conduction heat transfer resulted from both LTNE and classical theories. The Finite element based numerical technique is employed to formulate the problem. Finally the numerical results of transient change in temperature, pore pressure and stresses are presented and discussed.

2.Formulation of the problem

Biot's (1941) poroelasticity has been extended to model the thermo-poroelasticity in which the assumption of local thermal equilibrium (classical theory) has been used. The local thermal non-equilibrium concept, on the other hand, describes the heat interaction between the rock and fluid. In this theory (LTNE) the temperature of the rock depends on the thermal conduction and heat transfer between rock and fluid whereas temperature of fluid depends on the thermal conduction, convection and heat transfer between rock and fluid. The transient change in temperatures of rock and fluid are governed by the following equations:

$$(1-\phi_o)\rho_s c_s \frac{d\theta_s}{dt} = (1-\phi_o)k_{Ts}\nabla^2\theta_s + h_{\text{int}}(\theta_f - \theta_s) \qquad (1)$$

$$\phi_o \rho_f c_f \frac{d\theta_f}{dt} = \phi_o k_{Tf}\nabla^2\theta_f - (1-\phi_o)\rho_f c_f \nabla(J_f \theta_f) - h_{\text{int}}(\theta_f - \theta_s) \qquad (2)$$

Where θ_s and θ_f are the rock and fluid temperatures, ϕ_o is the rock porosity, k_{Ts} and k_{Tf} are thermal conductivities of the rock and fluid respectively. Also c_s and c_f are the specific heat capacity of rock and fluid, h_{int} is the rock-fluid interface heat transfer coefficient and ∇^2 the Laplacian operator. ρ_S and ρ_f are rock and fluid densities. The weighted average of temperature variations between rock and fluid is assumed as follow:

$$\theta_{avg} = (1-\phi_0)\theta_s + \phi_0\theta_f \qquad (3)$$

The obtained average temperature of the system is used to study the rock displacement and pore pressure. Thus, governing equations of non-isothermal thermo-poroelasticity can be expressed as (Kurashige 1989):

$$\dot{\sigma}_{ij} = (K - \frac{2G}{3})\dot{\varepsilon}_{ij}\,\delta_{ij} + 2G\dot{\varepsilon}_{ij} - \alpha\,\dot{p}\,\delta_{ij} + \gamma_1\dot{\theta}_{avg}\,\delta_{ij} \qquad (4)$$

$$\alpha \dot{\varepsilon}_{ii} + Q' \dot{p} - \gamma_2 \dot{\theta}_{avg} = \frac{k}{\mu} \nabla^2 p \qquad (5)$$

Where, K, G and α are bulk, shear modulus and Biot coefficient respectively. Also $\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{ij}$ are the components of the total stress and strain tensors differentiated based on time. p is pore pressure, k is the intrinsic permeability and μ is the fluid viscosity. The other parameters in governing equations are:

$$Q = \frac{(\alpha - \phi_0)}{K_S}, Q' = \left(Q + \frac{\phi_0}{K_f} \right) \qquad (6)$$

$$\gamma_1 = K\alpha_m, \gamma_2 = \alpha\alpha_m + (\alpha_f - \alpha_m)\phi_0 \qquad (7)$$

Where, α_m and α_f are thermal expansion coefficient of rock and fluid respectively and K_S is the rock bulk modulus.

3. Model description and Solution strategy

Model description

Model geometry includes a wellbore and the surrounding formation. Spatial discretization (FEM mesh) of the model geometry is shown in **Fig. 1**. Note that only a quarter of the model has been discretized by taking advantage of symmetry. 8-noded isoparametric quadrilateral elements have been used for rock displacement and 4-noded for pore pressure and temperature. Compressive stress is considered positive and tensile stress negative. Plane strain hypothesis and instantaneous drilling are used. The maximum (σ_H) and minimum (σ_h) horizontal stresses are aligned with the x and y axes respectively.

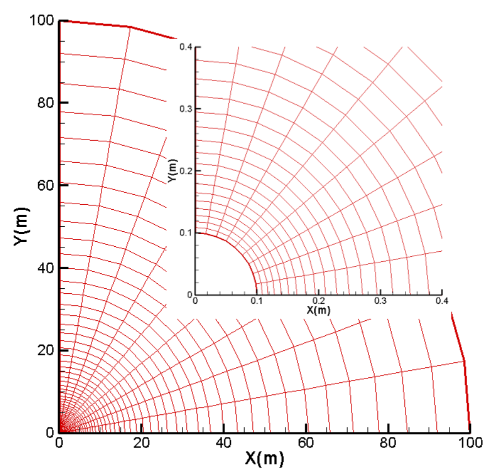

Fig. 1. Spatial discretization (FEM mesh of the model geometry)

Solution strategy

To solve the non-linear system of equations derived in this study, first the temperature equations for rock and fluid systems are solved separately by using Characteristic Galaerkin discretization technique. Then the rock displacement and pore pressure are solved simultaneously based on obtained temperatures. The direct iteration technique is then employed to achieve the exact solution of temperatures, pore pressure and rock displacement for each time step. Super convergent patch recovery method (Zienkiewicz and Zhu 1992; Boroomand and Zienkiewicz 1997) is used to evaluate nodal stress tensors from the numerical results of nodal displacement. Numerical results were validated against appropriate analytical solutions presented by Ekbote (2002) and Wang (2003). Injecting fluid and reservoir properties used in this study are presented in **Table 1** unless stated otherwise.

Table 1: Input data used in this study

Bulk Young's modulus	14.4 GPa
Drained Poisson's ratio	0.2
Porosity	0.2
Undrained Poisson's ratio	0.4
Initial reservoir pressure	12 MPa
Mud pressure	25 MPa
Fluid bulk module	2.5 GPa
Wellbore radius	0.1 m
Solid bulk module	48.2 GPa
Fluid viscosity	3×10^{-4}Pa.s
Fluid thermal conductivity	0.6 W/mK
solid thermal conductivity	2.4 W/mK
Fluid volumetric heat capacity	4.186×10^{7}J/m3.K
Solid volumetric heat capacity	1.547×10^{7}J/m3.K
Formation permeability	$9.869\times10^{-16}m^{2}$
Skempton coefficient	0.915
Heat transfer coefficient	50 W/m2.K
Thermal expansion coefficient of fluid	$1.0\times10^{-4}\ {}^{o}K^{-1}$
Thermal expansion coefficient of solid	$1.0\times10^{-5}\ {}^{o}K^{-1}$

4. Results and Discussion

In this study, it is assumed that the cold water is injected into a reservoir with relatively hot initial rock temperature. In order to show the effect of LTNE theory on pore pressure, temperature and stresses around the wellbore two cases are considered: LTNE and Classical theory. Also two states of isotropic in-situ stresses, $\sigma_H = \sigma_h$ (state A) and anisotropic in-situ stresses, $\sigma_H > \sigma_h$ (state B) with the assumption of $P_w > P_f$ (wellbore fluid pressure is higher than formation pore pressure) are considered to analyze the change in pore pressure and stresses.

Isotropic state of stress (State A): In this case, the maximum and minimum horizontal in-situ stresses are set to be 35MPa ($\sigma_H = \sigma_h = 35MPa$). **Fig.2** shows the temperature distribution along x axis after 10mins for both cases: LTNE and classical theory. The corresponding temperature of rock and fluid are also included. As it can be observed from this figure there is a significant difference in temperature distribution resulted from the classical theory and

LTNE (weighted average). The temperature profile obtained from classical theory is closer to fluid phase temperature rather than the weighted average temperature of the system. This is due to fact that the formulation of classical theory gives more preference to the fluid phase than the rock phase. It is also seen that the fluid temperature is lower than that of rock temperature for all radial positions. For example, the rock temperature is 450°K and fluid temperature is 425°K at the same radial position of 0.15m away from the wellbore wall which is expected because of time dependent heat transfer between fluid and rock. The heat transfer depends greatly on the coefficient of heat transfer between fluid and rock.

The results of temperature distribution along x axis after 4.5hrs for both cases: LTNE and classical theory are presented in **Fig. 3**. The similar observations to those presented in Fig.2 can be made in Fig.3. The temperature change due to LTNE, however, is more pronounced at late time (4.5hrs) compared to early time (see Fig. 2). This means that even after continuous injection of cold water for 4.5 hours into the hot formation the fluid and rock have not reached thermal equilibrium near wellbore area and the difference between the temperature of the rock and fluid has further increased.

The pore pressure distribution along x-axis at early (after 10mins) and late time steps (after 4.5hrs) is presented in **Fig.4** for both cases: LTNE theory and classical theory. From this figure it can be seen that pore pressure is not affected when LTNE is considered compared to classical theory at both early (10mins) and late times (4.5hrs) as they overlap each other. This is due to the fact that the difference in temperature change, obtained from both theories, is not high enough to affect the pore pressure arising from the difference between thermal expansion coefficients of rock and fluid.

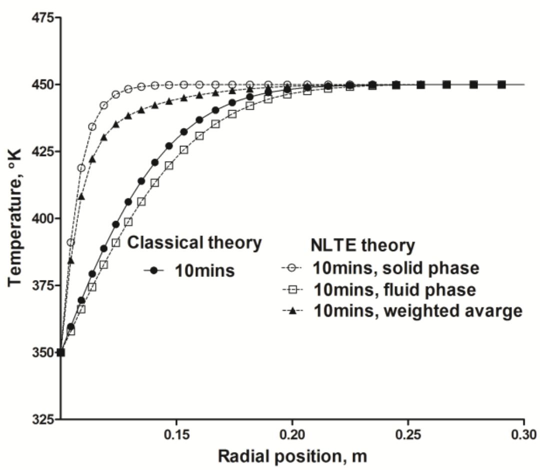

Fig. 2. Temperature distribution along x axis after 10mins for both cases: classical and LTNE theory.

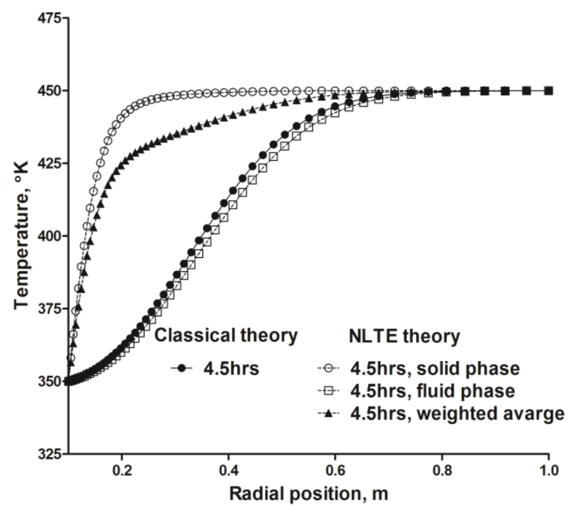

Fig. 3. Temperature distribution along x axis after 4.5hrs for both cases: classical and LTNE theory.

The effective radial stress distribution for both cases: LTNE and classical theory at early and later times (10mins and 4.5hrs) is presented in **Fig 5**. From the results of this study it can be observed that the extra compressive effective redial stress is concentrated near the wellbore wall when LTNE is considered. With the passing of time the increase in effective radial stresses becomes highlighted remarkably. This difference in effective radial stresses between

the classical and LTNE theory is mainly induced by thermal stresses coming from difference in heat transfer by LTNE and classical theory (see Figs. 2 and 3).
In **Fig. 6** the effective tangential stresses along x axes after 10mins and 4.5hrs for both cases: classic theory and LTNE are presented. From this figure it can be seen that the effective tangential stresses significantly increase when LTNE is considered. It can be pointed out

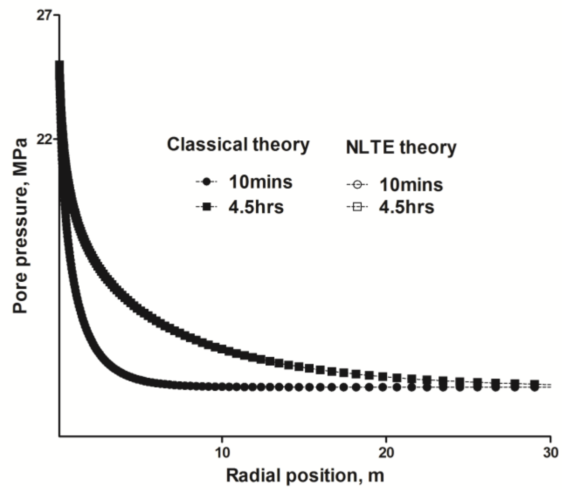

Fig. 4. Pore pressure distribution along x axis at two different time steps (10mins and 4.5hrs) for both cases: classical theory and LTNE theory.

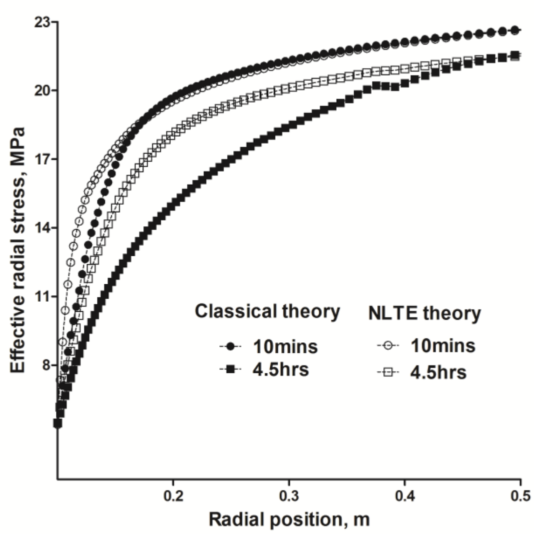

Fig. 5. Distribution of effective radial stress along x axis (10mins and 4.5hrs) for both cases: classical and LTNE theory.

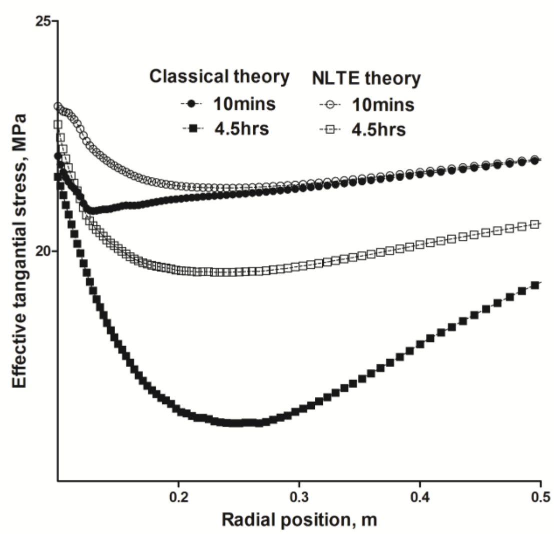

Fig. 6. Distribution of effective tangential stress along x axis (10mins and 4.5hrs) for both cases: classical and LTNE theory.

herein that the results obtain from classical theory overestimate the chance of tensile failure of the rock on the wellbore wall (hydraulic fracturing) during injection of cold water to the hot formation.
Anisotropic state of stress (State B): In this case, the maximum horizontal in-situ stress, σ_H is considered to be 45MPa whereas the minimum horizontal in-situ stress, σ_h is set to be

35MPa. **Figs. 7** and **8** show the 2D distribution of effective radial stresses after 4.5hrs for both classical and LTNE theory respectively. From these figures it can be observed that the radial effective stress becomes more compressive, as mentioned earlier, when LTNE is taken into account (Fig.8). Furthermore, this compression is particularly pronounced along minimum horizontal in-situ stress (y-axis), even though, the compression occurs all over the domain. For instance, the effective radial stress is about 20MPa at the radial position of 0.3m away from the wellbore wall when classical theory is considered (Fig.7) whereas it becomes about 25MPa at the same radial position for the LTNE theory (Fig.8).

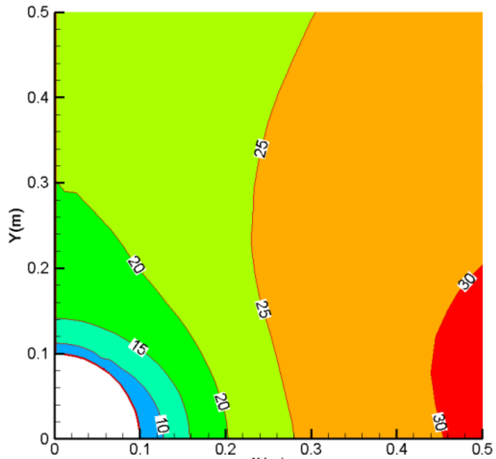

Fig. 7. 2D distribution of effective radial stress after 4.5hrs for the case of classical theory.

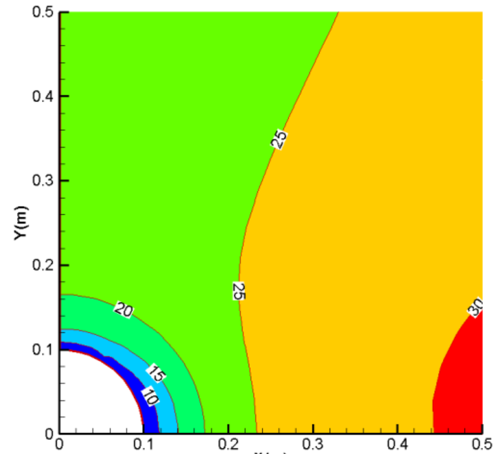

Fig. 8. 2D distribution of effective radial stress after 4.5hrs for the case of LTNE theory.

The 2D distribution of effective tangential stresses for both classical and LTNE theory are presented in **Figs. 9** and **10** respectively. The similar results can be observed for effective tangential stress compared to the effective radial stress (Figs.7 and 8). However, the LTNE theory seems to have a profound effect on tangential stress particularly along y axis. The tangential stress increases significantly from about 40MPa (classical theory, Fig.9) to 50MPa (LTNE theory, Fig.10) on the wellbore wall along y axis. Thus, it clearly suggests that the classical theory might not be suitable in stress analysis of relatively high permeable formations especially in the case of an anisotropic state of stresses.

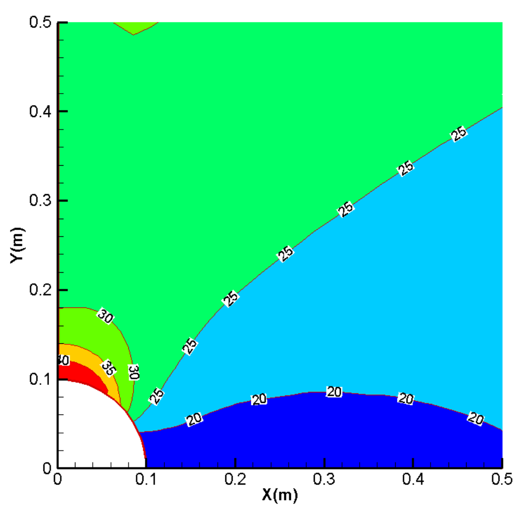

Fig. 9. 2D distribution of effective tangential stress after 4.5hrs for the case of classical theory.

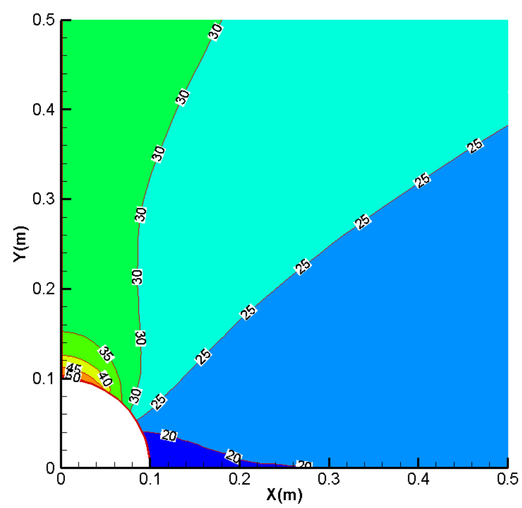

Fig. 10. 2D distribution of effective tangential stress after 4.5hrs for the case of LTNE theory.

5. Conclusions

In this study the change in temperature, pore pressure and effective stresses around a wellbore due to the response to classical and LTNE theories were studied. In order to do this, a finite element conductive-convective thermo-poroelastic model was developed. In LTNE formulation the assumption of instantaneous local thermal equilibrium between rock and fluid was replaced with the real time dependent heat transfer between rock and fluid. From the results of this study it was revealed that heat transfer is significantly affected when the LTNE theory is employed. The results showed that the classical theory is not fully applicable in relatively high permeable formations. It was also found that the effective radial stresses become more compressive during injection of cold water into a hot reservoir when LTNE is considered. When LTNE is taken into account the effective tangential stresses also remarkably increase on the wellbore wall which can prevent the problem of hydraulic fracturing, even though, the classical theory seems to over highlight the problem of hydraulic fracturing during cold water injection. Finally, the classical theory seems to fall short in analyzing the precise change in stresses particularly in an anisotropic state of stresses.

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